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Modeling of the Thermo-Deformation Behavior in a Laser-Hardened "Plunger-Barrel" Pair Considering Wear Effects

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Abstract

In this study, the elastic state arising at the interface between a laser-hardened plunger and its mating barrel was investigated. Laser surface hardening significantly enhances the material hardness and tribological performance, which leads to a more favorable distribution of temperature and deformation in the contact zone. In the analysis, the plunger surface was modeled as a periodic structure and examined within the framework of elasticity theory using the Kolosov–Muskhelishvili complex potentials method. The boundary conditions were simplified through Fourier series expansion, allowing the distribution of normal and frictional stresses within the contact area to be determined analytically. This approach enables a more accurate assessment of the influence of surface hardening on contact temperature and carries practical significance for improving the operational performance, reliability, and wear resistance of plunger-type components. The results of the research align with current scientific directions pursued at the "Department of Special Technologies and Equipment" of the Azerbaijan Technical University, and provide a solid theoretical foundation for industrial applications in the fields of tribology and contact mechanics.

Keywords: laser surface hardening, thermoelastic stress modeling, plunger–barrel wear, Fourier series expansion, Kolosov–Muskhelishvili method

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Yeyilmənin təsiri nəzərə alınmaqla lazerlə möhkəmləndirilmiş "plunjer-oymaq" cütündə termo-deformasiyanın modelləşdirilməsi

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Xülasə

Bu tədqiqat işində səthi lazer texnologiyası ilə möhkəmləndirilmiş plunjerin oymaqla təmasda olduğu zaman yaranan elastiklik vəziyyəti öyrənilmişdir. Lazerlə səthi möhkəmləndirmə nəticəsində materialın bərkliyi və triboloji göstəriciləri əhəmiyyətli dərəcədə artır, bu isə kontakt zonasındakı temperaturun və deformasiyaların daha əlverişli paylanmasına gətirib çıxarır. Tədqiqatda plunjerin səthi periodik struktur kimi modelləşdirilərək, elastiklik nəzəriyyəsi çərçivəsində Kolosov–Muskhelişvili kompleks potensialları metodu ilə təhlil edilmişdir. Sərhəd şərtləri Fourier sıraları vasitəsilə sadələşdirilmiş və nəticədə kontakt sahəsində normal və sürtünmə gərginliklərinin paylanması analitik üsullarla müəyyən edilmişdir. Bu yanaşma, səthi möhkəmləndirmənin kontakt temperaturuna olan təsirini daha dəqiq qiymətləndirməyə imkan verir və plunjer tipli hissələrin iş qabiliyyətinin və etibarlılığının artırılması, eləcə də onların yeyilməyə qarşı davamlılığının yüksəldilməsi baxımından praktik əhəmiyyət daşıyır. Tədqiqat nəticələri, xüsusilə Azərbaycan Texniki Universitetinin "Xüsusi Texnologiyalar və Avadanlıqlar" kafedrasında aparılan müasir triboloji və kontakt mexanikası sahəsindəki elmi istiqamətlərlə uzlaşır və sənaye tətbiqləri üçün mühüm nəzəri əsaslar formalaşdırır.

Açar sözlər: lazerlə səthi möhkəmləndirmə, istilik-elastiklik gərginliklərinin modelləşdirilməsi, plunjer–oymaq cütlüyündə yeyilmə, Furye sırası ilə genişlənmə, Kolosov–Muskhelishvili metodu.

Моделирование термодеформационного состояния пары «плунжер-втулка» с лазерными упрочнением с учётом износа

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Аннотация

В настоящем исследовании проанализировано напряжённо-деформированное состояние, возникающее в зоне контакта между лазерно упрочнённым плунжером и сопрягаемой втулкой. Лазерное упрочнение поверхности существенно повышает твёрдость материала и его трибологические характеристики, что приводит к более благоприятному распределению температуры и деформаций в контактной области. В рамках анализа поверхность плунжера моделировалась как периодическая структура и исследовалась на основе теории упругости с использованием метода комплексных потенциалов Колосова – Мусхелишвили. Граничные условия были упрощены путём разложения в ряд Фурье, что позволило аналитически определить распределение нормальных и касательных напряжений в контактной зоне. Такой подход обеспечивает более точную оценку влияния поверхностного упрочнения на контактную температуру и имеет практическое значение для повышения эксплуатационных характеристик, надёжности и износостойкости деталей плунжерного типа. Результаты исследования соответствуют современным научным направлениям кафедры «Специальные технологии и оборудование» Азербайджанского Технического университета и формируют прочную теоретическую базу для промышленных приложений в области трибологии и контактной механики.

Ключевые слова: лазерное упрочнение поверхности, моделирование термоупругих напряжений, износ в паре «плунжер – втулка», разложение в ряд Фурье, метод Колосова – Мусхелишвили.

Introduction

One of the main scientific and technical challenges in improving the reliability and durability of precision components in fuel pumps is surface strengthening. Tribological contact pairs subjected to high pressures and frictional effects – particularly the "plunger–barrel" interface and the impact-induced contact between the plunger's heel and the pusher bolt – often exhibit wear and degradation. These phenomena significantly reduce the durability of the components. To mitigate this problem, various surface hardening technologies are employed. One such method is laser-assisted diffusion metallization through the application of a strengthening paste [1–5].

The essence of this method lies in applying a specially formulated paste containing strengthening elements such as boron and chromium onto the surface of the component. This paste is then heated using a laser beam. Under the influence of laser energy, the surface is heated, and the elements within the paste diffuse into the surface layers of the material, forming a hard and wear-resistant coating. As a result, boride and chromide phases are generated on the surface. These coatings exhibit high hardness, resistance to oxidation and corrosion, as well as strong durability against friction [6, 7].

The "barrel – plunger" contact pair examined in this study represents a typical example of components that slide or move reciprocally under pressure. Within such a pair, friction between the contact surfaces leads to force interactions, contact pressure, and, consequently, wear phenomena. However, surfaces that have been pre-strengthened through laser-assisted diffusion metallization demonstrate higher resistance to this wear. Therefore, the application of such a technological treatment is of particular interest due to its impact on the wear intensity and the temperature – deformation state within the contact pair.

The wear process directly affects the size of the contact surface and the local distribution of contact pressure. This, in turn, requires an accurate evaluation of the stress and deformation state. In particular, modeling the tribological properties and stress distribution of coatings obtained through laser-induced boron–chromium diffusion enables the prediction of their mechanical durability [8, 9].

The aim of the work is to model the temperature–deformation state that arises in a "plunger–barrel" contact pair pre-hardened by laser-assisted paste application, taking into account the wear caused by friction. The results of the research are of significant importance for the design of highly reliable tribological pairs and for improving their durability.

Research methods

Let us consider the problem of determining the temperature in a "barrel – plunger" contact pair. During the plunger's numerous reciprocating movements, heat is generated as a result of its contact with the barrel [10, 11].

Since the frequency of the plunger's motion is sufficiently high, the problem is approached as a steady-state case. In the contact zone – the tribological interface – heat is generated on the outer surface of the barrel where it comes into contact with the plunger. As a result of this interaction, the temperatures of both the barrel and the plunger increase.

According to the theory of heat conduction, the boundary conditions across the cross-section of the plunger are defined as follows (in contact area (ICA), outside the contact area (OCA)):

$$\lambda\frac{\partial T}{\partial n} = -Q(\theta), (ICA). \quad (1)$$
$$\lambda\frac{\partial T}{\partial n} + \alpha(T - T_c) = 0, (OCA)$$

Here, T(r,θ) is the temperature function within the plunger; λ is the thermal conductivity of the plunger material; α is the heat transfer coefficient from the cylindrical surface of the plunger to the surrounding medium at temperature Tc; n is the normal to the outer contour of the plunger; and Q(θ) represents the intensity of the surface heat source acting on the plunger.

The thermal conductivity coefficients of the plunger material are assumed to be uniform along the axis, in the surrounding environment, and in the radial direction, and are considered independent of both coordinates and temperature.

For the intensity of the surface heat source in the friction zone:

$$Q(\theta) = \alpha_{m,n,1} f V p(\theta), \quad (2)$$

Here, f is the friction coefficient of the contact pair; V is the average circumferential velocity of the plunger relative to the barrel; p(θ) is the specific contact pressure on the friction surface; and $\alpha_{m,n,1}$ is the heat partition coefficient for the plunger, where $\alpha_{m,n,1}$ = 1 - $\alpha_{m,n}$.

The outer contour of the plunger is assumed to be circular. It is well known that the actual surface of the plunger is never perfectly smooth and inevitably contains irregularities as a result of the technological machining process.

The plunger is described in a polar coordinate system rθ, with the origin of the coordinates taken at the center of circle L, and with a radius denoted as R_1.

Let us consider a specific realization of the plunger's rough surface. Although the dimensions of the surface irregularities are very small, they have a significant impact on the temperature field in regions close to the surface, as well as on various operational characteristics of the contact in tribotechnical pairs.

Taking into account the excessive temperature:

$$t = T - T_c, \quad (3)$$

Let us represent the temperature in the plunger as an expansion in terms of a small parameter.

$$t = t^{(o)} + \varepsilon t^{(1)} + \cdots, \quad (4)$$

Here, for simplicity, terms of the small parameter ε of order higher than the first are neglected. In this case, $t^{(o)}$, $t^{(1)}$ represent the temperature in the zeroth and first approximations, respectively. Each approximation satisfies the differential equation of heat conduction (3). The values of the temperature components at r=ρ(θ) are obtained by expanding the temperature function in a series near r=R_1.

$$t^{(o)}_{/r=\text{p}} = t^{(o)}_{/r=R_1} + \varepsilon \frac{\partial t^{(o)}}{\partial r}_{/r=R_1} \cdot H_1(\theta) + \cdots. \quad (5)$$
$$t^{(1)}_{/r=\text{p}} = t^{(1)}_{/r=R_1} + \varepsilon \frac{\partial t^{(1)}}{\partial r/_{r\,=\,R_1}} \cdot H_1(\theta) + \cdots;$$

The boundary conditions of the heat conduction problem, accurate up to the first-order small parameter, are given as follows:

For the zeroth approximation:

$$\lambda\frac{\partial t^{(o)}}{\partial r} = -Q^{(o)}(\theta), (ICA), \quad (6)$$
$$\lambda\frac{\partial t^{(o)}}{\partial r} + \alpha t^{(o)} = 0, (OCA)$$

Where $Q^{(0)}(\theta) = \alpha_{m,n,1} f V p^{(0)}(\theta)$

For the first approximation:

When r = R_1,

$$\frac{\partial t^{(1)}}{\partial r} = -\frac{Q^{(1)}(\theta)}{\lambda} - \frac{\partial^2 t^{(0)}}{\partial r^2} H_1(\theta), \text{(ICA)} \quad (7)$$

$$\lambda \frac{\partial t^{(1)}}{\partial r} + \alpha t^{(1)} = \left[\alpha \frac{\partial t^{(0)}}{\partial r} - \lambda \frac{\partial^2 t^{(0)}}{\partial r^2}\right] H_1(\theta),$$

(OCA).

Where $Q^{(1)}(\theta) = \alpha_{m,n,1} f V p^{(1)}(\theta)$.

In each approximation, we seek the solution of the differential equation of heat conduction theory using the method of separation of variables [12].

$$t^{(0)} = \Phi^{(0)}(\theta) f^{(o)}(r);$$

$$t^{(1)} = \Phi^{(1)}(\theta) f^{(1)}(r), \quad (8)$$

For the temperature to be single-valued, the functions $\Phi^{(0)}$ and $\Phi^{(1)}$must be periodic. By substituting $t^{(0)}$ into the heat conduction equation (8), we obtain:

$$\frac{r^2 f^{(o)''} + r f^{(o)'}}{f^{(o)}} = -\frac{\Phi^{(0)''}}{\Phi^{(0)}} = \lambda^2, \quad (9)$$

Equation (9) yields two ordinary differential equations.

$$\Phi^{(0)''} + \lambda^2 \Phi^{(0)} = 0, \quad (10)$$

$$r^2 f^{(o)''} + r f^{(o)'} - \lambda^2 f^{(o)} = 0, \quad (11)$$

The general solution of equation (10) will be as follows:

$$\Phi^{(0)} = \mathrm{A} \cos\lambda\theta + \mathrm{B} \sin\lambda\theta, \quad (12)$$

And the solution (11) will be written in the following form:

$$f^{(0)} = C_1 r^{\lambda} + C_2 r^{-\lambda}, \quad (13)$$

Since the temperature at any point of the plunger must remain finite, the second term in equation (13) vanishes, i.e., C_2=0.

The temperature in the zeroth-order approximation will be expressed in the following form:

$$t^{(0)} = \\ = \sum_{n=0}^{\infty} r^n \left(C_1^{(n)} \cos n\theta + C_2^{(n)} \sin n\theta\right), \quad (14)$$

The constants $C_1^{(n)}$ and $C_2^{(n)}$ are determined from the boundary conditions given in equation (7).

After determining the function $t^{(0)}$, we compute the right-hand side of the boundary condition (8).

In the first-order approximation of the heat conduction theory problem, the solution is obtained in a manner similar to the zeroth-order approximation.

$$t^{(1)} = \\ \sum_{n=0}^{\infty} r^n \left(C_{11}^{(n)} \cos n\theta + C_{21}^{(n)} \sin n\theta\right), \quad (15)$$

The constants $C_{11}^{(n)}$ and $C_{21}^{(n)}$ are easily determined from the boundary conditions (8).

Based on the obtained formulas, the excess temperature on the surface of the plunger up to the first-order small parameter with respect to ε is given by:

$$t_* = t/_{r=\rho} = \frac{t(\theta)}{r = R_1} + \\ +\varepsilon\left[\frac{\partial t^{(0)}}{\partial r} H_1(\theta) + t^{(1)}\right]_{/r=R_1}, \quad (16)$$

Therefore, based on each predefined processed surface profile, it is possible to analyze the temperature state of the plunger using the obtained relations.

It is known that it is more appropriate to describe the surface of the plunger statistically, where the function $H_1(\theta)$ is considered as realizations of random values at various cross-sections [13]. A stationary random function in the interval [0,2π] is represented by a canonical expansion in the form of a Fourier series, where the coefficients of the series are uncorrelated random variables with zero mathematical expectations and variances D_k.

In such an approach, the temperature t_* acts as a random variable, for which the mathematical expectation and variance are determined using standard methods. Significant thermal stresses and structural changes in the material of surface layers occur in the components of the contact pair.

Therefore, under thermo-mechanical loading conditions, the material of the surface and subsurface layers of the plunger's friction element is investigated. Such an analysis is crucial for developing criteria for selecting the optimal design parameters and materials of the contact pair components.

The problem of determining the temperature of the plunger in the tribotechnical pair was discussed in detail in our previous studies. The plunger is assigned to the polar coordinate system r(θ), with the origin of the coordinates located at the center of the circle L and a radius R1. The external contour of the plunger is assumed to be circular.

Let us consider a specific realization of the plunger's rigid surface. The contour boundary of the plunger is assumed to take the form L′.

The boundary conditions for thermal stresses on the cross-sectional surface of the plunger are defined as follows.

When $\sigma_n^T = 0;\ \tau_{nt}^T = 0; r = \rho(\theta)$, (17)

For convenience in future notation, we will omit the superscript on "T". We seek the stresses and displacements in the plunger in the form of expansions with respect to the small parameter ε, where, for simplicity, we neglect terms involving powers of ε higher than the first.

Here, $\sigma_r^{(0)}, \sigma_\theta^{(0)}, \tau_{r\theta}^{(0)}, v_r^{(0)}, v_\theta^{(0)}$ represent the stresses and displacements in the zeroth approximation, while $\sigma_r^{(1)}, \sigma_\theta^{(1)}, \tau_{r\theta}^{(1)}, v_r^{(1)}, v_\theta^{(1)}$ correspond to the stresses and displacements in the first approximation.

Each approximation must satisfy the system of differential equations of thermoelasticity theory [14].

When r=ρ(θ), the values of the components of the stress tensor are obtained by expanding the stress expressions into a series around r=R_1.

The boundary conditions for the thermoelastic stress problem in the zeroth approximation are as follows.

When $r = R_1\ \sigma_r^{(0)} = 0; \quad \tau_{r\theta}^{(0)} = 0,$ (18)

For the first approximation, we obtain:

When $r = R_1\ \sigma_r^{(1)} = \mathrm{N}; \quad \tau_{r\theta}^{(1)} = \mathrm{T},$ (19)

Here, the functions N and T are defined in advance by the formulas.

In each approximation, the solution of thermal deformations is obtained using the thermoelastic potential.

In the problem under consideration, the thermoelastic potential in the zero approximation is defined by the following equation.

$$\frac{\partial^2\Phi^{(0)}}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi^{(0)}}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi^{(0)}}{\partial\theta^2} = \frac{1+\mu}{1-\mu}\alpha t^{(0)}, \quad (20)$$

The temperature function $t^{(0)}(r,\theta)$ is expressed in the form of a Fourier series (see formula (15)).

The solution to equation (20) for the thermoelastic potential is sought in the following form:

$$\Phi^{(0)} = \sum_{n=0}^{\infty}\left[\Phi_n^{(0)}\cos n\theta + \Phi_n^{*(0)}\sin n\theta\right], \quad (21)$$

By substituting equation (21) into the differential equation (20), we obtain ordinary linear homogeneous second-order differential equations for the functions $\Phi_n^{(0)}$ (r) and $\Phi_n^{*(0)}(r)$.

$$\frac{d^2\Phi_n^{(0)}}{dr^2} + \frac{1}{r}\frac{d\Phi_n^{(0)}}{dr} - \frac{n^2}{r^2}\Phi_n^{(0)} = \frac{1+\mu}{1-\mu}\alpha F_n^{(0)}, \quad (22)$$

$$\frac{d^2\Phi_n^{*(0)}}{dr^2} + \frac{1}{r}\frac{d\Phi_n^{*(0)}}{dr} - \frac{n^2}{r^2}\Phi_n^{*(0)} = \frac{1+\mu}{1-\mu}\alpha F_n^{*(0)}$$

It is appropriate to search for particular solutions of equations (22) using the method of variation of parameters.

$$\Phi_n^{(0)} = \frac{1+\mu}{1-\mu}\frac{\alpha}{2n}\left[r^n\int_r^R\left(F_n^{(0)}(\rho)\rho^{1-n}d\rho\right)\right], \quad (23)$$

$$\Phi_n^{*(0)} = \frac{1+\mu}{1-\mu}\frac{\alpha}{2n}\left[r^n\int_r^R\left(F_n^{*(0)}(\rho)\rho^{1-n}d\rho\right)\right]$$

according to the formulas:

$$\overline{\sigma}_r^{(0)} = -2G\left(\frac{1}{r}\frac{\partial\Phi^{(0)}}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi^{(0)}}{\partial\theta^2}\right), \qquad (24)$$

$$\overline{\tau}_{r\theta}^{(0)} = 2G\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Phi^{(0)}}{\partial\theta}\right); \quad \overline{\sigma}_\theta^{(0)} = -2G\frac{\partial^2\Phi^{(0)}}{\partial r^2};$$

We calculate the corresponding components of the stress tensor.

The obtained stresses (24) do not satisfy the boundary conditions (18) for the thermoelastic stress state. This discrepancy arises due to the influence of the non-uniform temperature field of the contact pair within the plunger. Therefore, a second auxiliary stress state is introduced based on the boundary condition.

$$\overline{\overline{\sigma}}_r^{(0)}, \overline{\overline{\sigma}}_\theta^{(0)}, \overline{\overline{\tau}}_{r\theta}^{(0)},$$

need to find.

When $r = R_1$:

$$\overline{\overline{\sigma}}_r^{(0)} = -\overline{\sigma}_r^{(0)}, \quad \overline{\overline{\tau}}_{r\theta}^{(0)} = -\overline{\tau}_{r\theta}^{(0)}, \qquad (25)$$

To solve the boundary value problem (25), we use the method of N. I. Muskhelishvili [15-16].

$$\varphi^{(0)}(z) = \sum_{k=1}^{\infty} a_k z^k;$$

$$\psi^{(0)}(z) = \sum_{k=0}^{\infty} a'_k z^k, \qquad (26)$$

Using formula (24) and the Kolosov–Muskhelishvili formulas, we determine the stresses in the zeroth approximation.

$$\sigma_r^{(0)} = \overline{\sigma}_r^{(0)} + \overline{\overline{\sigma}}_r^{(0)} = 0;$$

$$\sigma_\theta^{(0)} = \overline{\sigma}_\theta^{(0)} + \overline{\overline{\sigma}}_\theta^{(0)} = 0;$$

$$\tau_{r\theta}^{(0)} = \overline{\tau}_{r\theta}^{(0)} + \overline{\overline{\tau}}_{r\theta}^{(0)} = 0, \qquad (27)$$

As expected, all stress components $\sigma_r^{(0)}, \sigma_\theta^{(0)}, \tau_{r\theta}^{(0)}$ are equal to zero due to the direct compatibility of the field and the absence of internal heat sources within the domain.

$$\overline{v}_r^{(0)} = \frac{\partial\Phi^{(0)}}{\partial r}; \quad \overline{v}_\theta^{(0)} = \frac{1}{r}\frac{\partial\Phi^{(0)}}{\partial\theta}, \qquad (28)$$

$$\overline{\overline{v}}_r^{(0)} + i\overline{\overline{v}}_\theta^{(0)} =$$

$$= \frac{e^{-i\theta}}{2G}\left[ae\varphi^{(0)}(z) - z\overline{\varphi'^{(0)}(z)} - \overline{\psi^{(0)}(z)}\right]$$

$$v_r^{(0)} = \overline{v}_r^{(0)} + \overline{\overline{v}}_r^{(0)}; \; v_\theta^{(0)} = \overline{v}_\theta^{(0)} + \overline{\overline{v}}_\theta^{(0)}$$

In the zeroth approximation, only the displacements related to temperature are determined.

In the first approximation, the thermoelastic potential displacements are defined by the corresponding equation.

$$\Delta\Phi^{(1)} = \frac{1+\mu}{1-\mu}\alpha t^{(1)}, \qquad (29)$$

The temperature function $t^{(1)}(r,\theta)$ is assumed in the form of a Fourier series (see formula (15)).

The thermoelastic potential for equation (29) is sought in the following form.

$$\Phi^{(1)} =$$

$$= \sum_{n=0}^{\infty}\left[\Phi_n^{(1)}\cos n\theta + \Phi_n^{*(1)}\sin n\theta\right], \qquad (30)$$

The subsequent solution procedure closely resembles the method used to find the solution in the zero-order approximation, with evident modifications.

All components of the stress tensor in the plunger, $\sigma_r^{(1)}, \sigma_\theta^{(1)}, \tau_{r\theta}^{(1)}$, are equal to zero. The displacement vector components $v_r^{(1)}$ and $v_r^{(1)}$ are determined by formulas similar to equation (28), with evident modifications.

Conclusion

In this study, the thermal distribution on the surface of the plunger and the resulting thermal stresses were analyzed using analytical methods. The differential equations of heat conduction and thermoelasticity theory were solved using the method of separation of variables, and the zero-order and first-order approximations of the temperature field were obtained analytically.

The temperature function on the plunger surface was expressed in the form of a Fourier series, and the temperature components were determined in accordance with the boundary conditions. Additional conditions were applied to ensure the uniqueness and physical validity of the solution, particularly ensuring the finiteness of temperature as $r \to 0$.

Due to the non-uniform distribution of the heat source, the resulting thermal stresses do not fully satisfy the initial boundary conditions. This inconsistency leads to additional mechanical loading and stress in the surface and subsurface layers of the contact pair. To resolve this, a secondary auxiliary stress state was formulated, and the general stress components were adjusted to satisfy all boundary conditions.

In both the zero-order and first-order approximations, the thermoelastic potential and the components of the stress tensor were determined using the Kolosov–Muskhelishvili complex function method. The stress analysis revealed that non-uniform thermal distribution on the plunger surface induces significant localized thermal stresses, potentially causing structural changes in the material.

Thus, the presented approach based on heat conduction and thermoelasticity theory enables a more accurate and realistic modeling of the temperature and thermal stress distribution on the surface of the plunger in the “barrel–plunger” contact pair. The obtained results serve as a scientific basis for the optimal design of tribological contact elements, material selection, and improved resistance to thermal-mechanical loading.

Conflict of Interests

The authors declare there is no conflict of interests related to the publication of this article.

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