Pricing Asian Options with Monte-Carlo Method

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Abstract

The paper is the Computational Finance project on pricing Asian options with the Monte-Carlo method. The certain discounted payoff formula is given under the risk-neutral density. The author reviews the arithmetic sampling payoff formula for fixed and floating strikes; the geometric sampling payoff formula for the fixed and floating strikes. In both cases the author use the Euler-Maruyama scheme for simulating the underlying stock price using the following set of data: today's stock price $S0 = 100;$ strike $K = 100$; time to expiry $(T - t) = 1$ *year*; volatility $\sigma = 20\%$; constant risk-free interest rate $r = 5\%$. As a result following is produced: outline of the numerical procedure; appropriate tables, comparisons and error graphs; analysis of observations and problems encountered.

Keywords: asian option, arithmetic sampling, geometric sampling, current price, fixed and floating strikes, time to expiry, volatility, interest rate.

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Asiya seçimlərinin Monte-Carlo metodu ilə qiymətləndirilməsi

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Annotasiya

Məqalədə Monte-Carlo metodu ilə Asiya seçimlərinin qiymətlərini müəyyənləşdirməyə kömək edən bir Hesablama Maliyyəsi layihəsi aparılıb. Güzəştli ödəmə üçün xüsusi düstur neytral risk sıxlığında verilib. Müəllif fiksasiya olunan və üzən məzənnələr üçün ödənişlərin cəbri seçmə formulunu araşdırıb; fiksasiya olunan və üzən məzənnələr üçün həndəsi seçmə ilə düstur alıb. Hər iki halda da müəllif aşağıdakı verilənlər bazasından istifadə edərək əsas səhm qiymətini modelləşdirmək üçün Eyler-Maruyama sxemindən istifadə edib: bugünkü səhm qiyməti $S_0 = 100$; məzənnə $K = 100$; sona çatma müddəti $(T - t) = 1$ *il*; dəyişkənlik σ = 20%; daimi risksiz faiz dərəcəsi r = 5%. Nəticədə ədədi prosedurun sxemi; müvafiq cədvəllər, müqayisələr və səhvlərin qrafikləri; müşahidələrin və yaranan problemlərin təhlili əldə edilib.

Açar sözlər: asiya seçimi, cəbri nümunə, həndəsi nümunə, cari qiymət, fiksasiya olunan və üzən məzənnə, bitmə müddəti, dəyişkənlik, faiz dərəcəsi.

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Ценообразование азиатских опционов по методу Монте-Карло

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Аннотация

Статья представляет собой исследование Computational Finance по ценообразованию азиатских опционов с помощью метода Монте-Карло. Определенная формула дисконтированной выплаты дана при нейтральной к риску плотности. Автор рассматривает формулу арифметической выборки выплат для фиксированных и плавающих страйков, а также формулу выигрыша с геометрической выборкой для фиксированных и плавающих страйков. В обоих случаях автор использует схему Эйлера-Маруямы для моделирования базовой цены акции с использованием определенного набора данных: сегодняшняя цена акции $S_0 = 100$; страйк $K= 100$; срок до истечения срока $(T - t) = 1$ *год*; летучесть $\sigma = 20\%$; постоянная безрисковая процентная ставка $r = 5\%$. Исследована схема численной процедуры, а также даны соответствующие таблицы, сравнения и графики ошибок, анализ наблюдений и возникших проблем.

Ключевые слова: азиатский опцион, арифметическая выборка, геометрическая выборка, текущая цена, фиксированные и плавающие страйки, время до истечения срока, волатильность, процентная ставка.

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Introduction

Asian options are a type of exotic options with a payoff that depends on the average price of the underlying asset over the specified period. As such an Asian option might be less costly than an equivalent vanilla contract. Since it is quite hard to manipulate the average price over time than a single stock price, the Asian options are obvious choice for the commodity and energy market participants.

Asian options were first priced by David Spaughton and Mark Standish of Bankers trust. They developed the first commercially used pricing formula for options based on the average price of crude oil in 1987 while working in Tokyo, Japan. Hence, there is a name "Asian".

As an exotic option, the Asian tail offers further protection against risk by reducing exposure to sudden movements in the underlying asset just before expiry.

One disadvantage of Asian options is that their prices are very hard to compute using standard techniques. Unlike European options, which can be priced using the classic Black-Scholes formula, there is no analytical formula for pricing an Asian option when the underlying asset is assumed to have a lognormal distribution.

Option Payoff

By taking the payoff formula for the vanilla contract and replacing the current asset price with its average we get the formula forth average rate option payoff

Call max $(A - K, 0)$

Put max $(K - A, 0)$, (1)

where A – the average price of the underlying asset over some specified period; K – the strike price.

This payoff formula allows to lock in the price of the underlying asset over extended period time.

Type of Sampling

The setting the payoff formula for the Asian call and put options, we must look into input parameters. The strike price is usually given by the contract. In our project we will look into two types of strikes: fixed and floating. The second parameter is the average of the underlying asset price. Here, we are going to look into different types of averaging the price of underlying asset. The averaging for an Asian option depends on two factors:

✤ how the data points are combined to form an average;

✤ what data points to be used.

The first one points toward the type of sampling, such as the arithmetic and geometric sampling. The second one defines the data set, such as all quoted prices and the subset from the global data.

By definition, the *arithmetic average* A_a of the price is the sum of all the constituent prices equally weighted, divided by the total number of prices used

$$A_a^d = \frac{1}{N}\sum_{k=1}^{N} S_k, \quad A_a^c = \frac{1}{t}\int_0^t S(\tau)d\tau \qquad (2)$$

The *geometric average* A_g is the exponential of the sum of all the logarithms of the constituent prices equally weighted, divided by the total number of prices used [Wilmott, 2016].

$$A_a^d = exp\left(\frac{1}{N}\sum_{k=1}^{N} logS_k\right),$$
$$A_g^c = exp\left(\frac{1}{t}\int_0^t logS(\tau)d\tau\right) \qquad (3)$$

With respect of what data points to be used, we can identify the continuously samples average, and the discretely sampled average.

The *continuously sampled average* A^c is called the sum of closely spaced prices taken over a finite time in such way, that average become integrals of the asset over the averaging period.

The *discretely sampled average* A^d is called the data points of the set of reliable closing prices over small time period.

In our project we will look at the continuous arithmetic and geometric sampling with fixed and floating strikes.

Solution methods

In order to solve our problem, we have two methods: the partial differential equation approach and the Monte Carlo method. Although, the partial differential method is faster and more flexible, it is much harder to code. The Monte Carlo method is much easier to code. Under the Monte Carlo method, the value of option is the present present value of the expected payoff under a risk neutral random walk

$$V(S,t) = E^{Q}\left[exp - \int_t^T r_\tau d\tau \text{Payoff}(S_T)\right] \quad (4)$$

The pricing algorithm is following [Wilmott, 2016]:

1. Simulate the risk-neutral random walk starting at today's value of the asset over the required time horizon. This gives one realization of the underlying price path;
2. For this realization calculate the option payoff;
3. Perform many more such realizations over the time horizon;
4. Calculate the average payoff over all realizations;
5. Take the present value of this average, this is the option value.

As part of the exercise we have chosen following parameters for pricing Asian call options: today's stock price S_0 = 100; strike *K=100*; time to expiry *(T-t)=1 year*; volatility σ = 20%; constant risk-free interest rate r = *5%.* We set the time-steps at 100, while varying then number of replications from 10 to 10^8.

In this report we consider the Asian call options based on their strike type and the sampling choice with following payoff formulas for each type (here, we replace for average with to avoid confusion with Arithmetic):

1. Arithmetic sampling: fixed strike

$$C_{afx} = max(V_a - K, 0), \quad (5)$$

2. Arithmetic sampling: floating strike

$$C_{afl} = max(S_T - V_a, 0), \quad (6)$$

3. Geometric sampling: fixed strike

$$C_{gfx} = max(V_g - K, 0), \quad (7)$$

4. Geometric sampling: floating strike

$$C_{gfl} = max(S_T - V_g, 0), \quad (8)$$

We use Maple as a primary choice of software to generate a path base on following formula

$$S_{t+\delta t} = S_t(1 + r\delta t + \sigma\phi\sqrt{\delta t}) \quad (9)$$

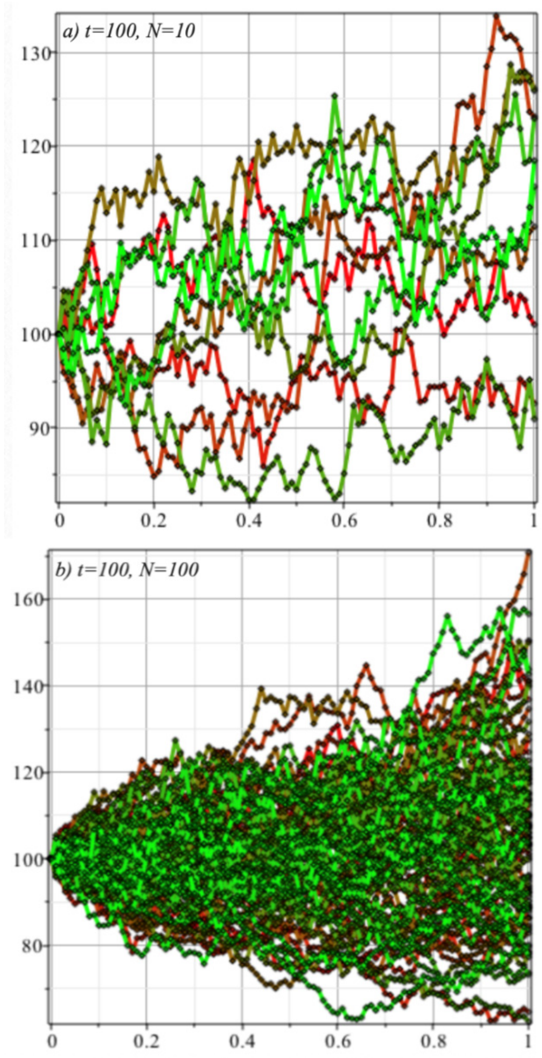

Figure 1. The MonteCarlo method for N replications

Figures 1 (a–b) compares of the replicated stock prices with the fixed t = 100 time-steps and varying number of replications N = 10 and N = 100. Having N = 100 and above ensures the normal distribution of the asset prices. Although, any values for $N > 10^8$ is beyond the scope of this work, since the limit capacity of the employed computer. We use the regular MACBOOKPro with Processor 3 GHz Intel Core i7 and Memory 16 GB 1600 MHz DDR3. All prices are rounded up to three decimal places, as if you pay cash, the more detail prices are provided in Workbook01.xls.

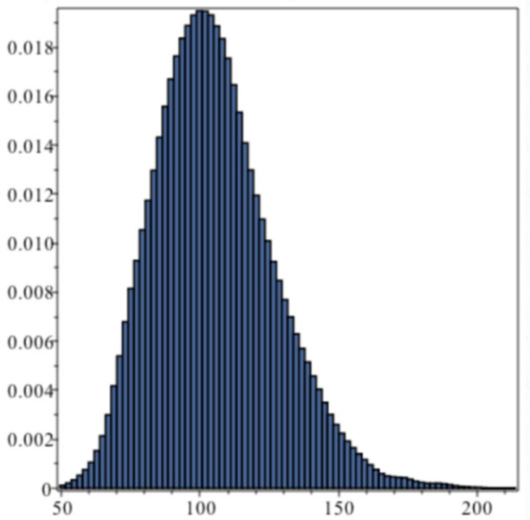

Figure 2. The MonteCarlo asset price distribution

The analytical solution of PDE for Asian Call Option provides us with the initial price as 5.763. We gonna use it as reference to find the price approximation for our simulation model.

The standard error is provides by the Maple Statistics package (library) based on formula.

$$\epsilon_N = \frac{\sigma}{\sqrt{N}} \qquad (10)$$

Table 1 shows he error factors in pricing Asian Options for each type of options.

Table 1. The error factor in pricing of Asian options (call-put)

Num-ber of simula-tions	Arithmetic				Geometric			
	Fixed		Floating		Fixed		Floating	
N	Call	Put	Call	Put	Call	Put	Call	Put
10	3.766	1.842	5.159	0.356	3.380	1.249	2.520	2.335
100	0.979	0.618	0.952	0.568	0.704	0.627	0.906	0.517
1000	0.270	0.182	0.279	0.166	0.248	0.187	0.315	0.172
10000	0.084	0.054	0.091	0.055	0.081	0.058	0.091	0.053
100000	0.026	0.017	0.029	0.017	0.025	0.018	0.030	0.017
1000000	0.008	0.005	0.009	0.005	0.008	0.006	0.009	0.005
10000000	0.003	0.002	0.003	0.002	0.003	0.002	0.003	0.002
100000000	0	0	0	0	0	0	0	0

We start our analysis by comparing prices for Asian Call Options at the money (S_t = K). Table 2 shows prices for Asian Options at $S_t = K$ = 100.

Table 2. The prices of Asian options (call-put) at the money

Number of Simulations	Arithmetic				Geometric			
	Fixed		Floating		Fixed		Floating	
N	Call	Put	Call	Put	Call	Put	Call	Put
10	7.445	2.533	5.960	1.838	6.418	1.251	3.149	5.555
100	4.783	4.299	6.299	3.900	6.102	4.231	5.159	3.073
1000	5.294	3.205	6.051	3.645	5.081	3.519	5.869	3.246
10000	5.881	3.334	5.856	3.410	5.568	3.423	6.050	3.182
100000	5.682	3.349	5.938	3.418	5.503	3.460	6.115	3.317
1000000	5.722	3.323	5.900	3.432	5.507	3.443	6.123	3.300
10000000	5.714	3.323	5.908	3.428	5.499	3.441	6.121	3.301
100000000	5.715	3.322	5.911	3.428	5.500	3.440	6.119	3.300

As we can see from Table 2, the prices become more stable after $N > 10^6$ replications. If you have to pick, then most expensive option that will be Geometric Floating Strike Asian Call Option. The cheapest is also Geometric but Fixed Strike Asian Call Option.

Next, we look at Asian Call Option out the money ($S_t < K$). Table 3 shows prices for Asian Options at $K > S_t = 90$.

As we can see from Table 3, the prices also become more stable after $N > 10^6$ replications. If you have to pick, then most expensive option that will be Geometric Floating Strike Asian Call Option. The cheapest is also Geometric but Fixed Strike Asian Call Option.

Next, we look at Asian Call Option in the money ($S_t > K$). Table 4 shows prices for Asian Options at $K < S_t = 110$.

As we can see from Table 4, the prices become more stable after $N > 10^6$ replications. If you have to pick, then most expensive option that will be Arithmetic Fixed Strike Asian Call Option. The cheapest is Arithmetic Floating Strike Asian Call Option.

Table 3. The prices of Asian options (call-put) out the money

Number of Simulations	Arithmetic				Geometric			
	Fixed		Floating		Fixed		Floating	
N	Call	Put	Call	Put	Call	Put	Call	Put
10	0.898	9.626	5.045	1.126	1.976	8.148	8.488	4.794
100	1.567	10.045	4.853	2.737	1.251	7.596	5.430	2.941
1000	1.824	8.780	5.351	3.280	1.424	9.113	5.727	3.072
10000	1.522	8.817	5.235	3.060	1.411	9.060	5.537	2.907
100000	1.528	8.886	5.331	3.065	1.415	9.073	5.512	2.974
1000000	1.537	8.886	5.304	3.083	1.413	9.072	5.514	2.973
10000000	1.536	8.892	5.317	3.085	1.413	9.070	5.512	2.972
100000000	1.536	8.892	5.317	3.085	1.413	9.070	5.512	2.972

By looking at these tables and comparing the rates of change of derivative per 10% move in the underlying up or down, we can say that Asian Options with Fixed Strike are more sensitive to the movement of the underlying. Meanwhile, Asian Option with Floating Strike, especially the Geometric Floating Strike Asian Options, act more like a damper by reducing the oscillation in underlying price.

The call-put parity doesn't hold in the standard definition. The asset price on the right hand side of equality actually represents the average underlying price.

Table 4. The prices of Asian options (call-put) in the money

Number of Simulations	Arithmetic				Geometric			
	Fixed		Floating		Fixed		Floating	
N	Call	Put	Call	Put	Call	Put	Call	Put
10	20.710	0.303	6.060	1.805	15.027	0.660	6.550	2.848
100	14.544	1.068	5.350	2.690	12.261	0.942	6.968	4.418
1000	12.963	0.881	6.302	3.529	12.731	0.865	6.668	3.707
10000	13.070	0.879	6.666	3.794	12.644	0.963	6.731	3.551
100000	12.990	0.860	6.538	3.754	12.674	0.938	6.756	3.629
1000000	12.994	0.862	6.503	3.778	12.689	0.926	6.722	3.624
10000000	13.006	0.860	6.505	3.770	12.703	0.929	6.728	3.631
100000000	13.005	0.861	6.502	3.770	12.706	0.929	6.731	3.631

Conclusion

Here are the major points of our research:

The Euler method generates the lognormal distributed asset prices.

The higher the number of repetitions the more accurate the price of derivative.

Asian Arithmetic Fixed Call Option is more sensitive to the price movement of underlying asset.

Asian Geometric Floating Call Option is less sensitive to the price movement of underlying asset.

The call-put parity doesn't hold in the standard definition. The asset price on the right hand side of equality actually represents the average underlying price.

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