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Sedimentation and Diffusion of Nanofluids in Rotating Systems of Metal Friction Elements of Brake *(Part II)*

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Abstract

For a capillary-porous body formed from nanoparticles, the following characteristics are determined: a differential distribution curve of pores and their total volume per unit volume of the body, the number of reduced cylindrical capillaries in a given cross section, and a capillary pressure curve. The latter are consistent with the thermodynamic relations of the displacement of the wetting phase. The regimes of fluid movement are established, when the accompanying forces are: inertia, capillary, gravitational. A mathematical description of diffusion-sedimentary processes in rotating metal friction elements of braking devices is given.

Keywords: band-shoe brake, friction pairs, brake pulley rim, nanoparticles, liquid, diffusion-sedimentation processes, molecular weight.

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Əyləclərin metal friksion elementlərinin fırlanan sistemlərinin nanomayelərində çökmə və diffuziya *(II hissə)*

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Xülasə

Məqalədə, nanohissəciklərdən ibarət kapilyar məsaməli cisim üçün aşağıdakı xüsusiyyətlər müəyyən edilmişdir: məsamələrin differensial paylanma əyrisi və onların cismin vahid həcminə düşən ümumi həcmi, verilmiş en kəsiyində verilmiş silindrik kapilyarların sayı, kapilyar təzyiq əyrisi. Verilənlər nəmlənmə fazasının yerdəyişmə prosesinin termodinamik əlaqələrinə uyğundur. Ətalət, kapilyar, cazibə qüvvələri zamanı mayenin hərəkət rejimləri müəyyən edilib. Əyləc qurğularının fırlanan metal sürtünmə elementlərində diffuziya-çökmə proseslərinin riyazi təsviri verilib.

Açar sözlər: lent-kündəli əyləc, sürtünmə cütü, əyləc qasnağı, nanohissəciklər, maye, diffuziya-çökmə prosesi, molekulyar çəki.

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Седиментация и диффузия в наножидкости во вращающихся системах металлических фрикционных элементов тормозов *(часть II)*

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Аннотация

Для капиллярно-пористого тела, образованного из наночастиц, определены следующие характеристики: дифференциальная кривая распределения пор и их суммарный объем в единице объема тела, количество приведенных цилиндрических капилляров в заданном сечении, кривая капиллярного давления. Последние согласуются с термодинамическими соотношениями процесса вытеснения смачивающей фазы. Установлены режимы движения жидкости, когда сопутствующими силами являются инерции: капиллярные, гравитационные. Приведено математическое описание диффузионно-седиментальных процессов во вращающихся металлических фрикционных элементах тормозных устройств.

Ключевые слова: ленточно-колодочный тормоз, пары трения, обод тормозного шкива, наночастицы, жидкость, диффузионно-седиментационные процессы, молекулярный вес.

Introduction

The results obtained are due to new approaches to the study of the properties of the system "nanocapillaries - nanofluid" in the chamber of the rim of the pulley of a band-shoe brake. When studying kinetic properties, this is primarily a rejection of established ideas about constant diffusion coefficients in a nanofluid, caused by the sedimentation of nanoparticles in a liquid.

The analysis of literary sources and the state of the problem are disclosed in the first part of the article. The main questions of the article: capillary-porous bodies formed from nanoparticles; diffusion-sedimentary processes in rotating systems; the discussion of the results.

The purpose of the work is to substantiate the performance of nanoparticles in capillary structures and in liquid in rotating brake systems.

Capillary porous bodies formed from nanoparticles. The main structural characteristic of capillary-porous bodies formed from nanoparticles is the pore distribution curve along its radius. The integral pore distribution curve characterizes the change in the relative pore volume *V** (ratio of pore volume to body volume) along the capillary radius *r*. Curve *V* = f(r)* starts from some value $r_{\min}$ (minimum capillary pore radius) and crosses the volume axisV^* with the value $r = r_{\max}$. Thus, within the limit $r_{\min} \leq r \leq r_{\max}$ relative volume *V** increases continuously with increasing *r*. If in some area $r_1 \leq r \leq r_2$ there is no such radius, then the curve *V* = f(r)* turns into a straight line parallel to the axis *r*. Addiction $dV^*/dr = f_v(r)$ is called differential pore distribution curve or differential equation of pore volume characteristic [1, 2]. This curve can be obtained from the integral curve *V* = f(r)* graphic differentiation method. The total volume of pores per unit volume of the body, equal to the porosity of the body, is determined by the relation

$$\Pi_v = V^*_{\max} = \int_{r_{\min}}^{r_{\max}} \frac{dV^*}{dr} dr = \int_{r_{\min}}^{r_{\max}} f_v(r) dr. \qquad (1)$$

Maximum liquid content $\omega_{\max}$ will be equal to:

$$\omega_{\max} = V^*_{\max} \rho_i = \Pi_V \rho_i, \qquad (2)$$

where ρ_l – liquid density.

The specific content of liquid, or its relative concentration (the mass of liquid in a porous body, referred to a unit mass of an absolutely dry body), is equal to:

$$u = \frac{1}{\rho_s} \omega = \frac{\rho_l}{\rho_s} \int_{r_{\min}}^{r} f_v(r) dr, \qquad (3)$$

where ρ_s - density of an absolutely dry porous body.

To calculate the flow density of a liquid passing through a porous body composed of nanoparticles, the concept of a differential curve of the surface porosity of a body is introduced. Let the cross-sectional area of the body be 1 cm^2, part of this area is occupied by capillaries (doors), the rest is the substance of the body itself. If all capillaries are filled with liquid, then the resistivity is:

$$u = \frac{\int_0^L \rho_l A_l dL}{\rho_s \cdot 1 \cdot L}, \qquad (4)$$

where A_l – the area occupied by the liquid, i.e. the area of all holes filled with liquid and located per 1 cm^2 of the body section; *L* – depth (thickness) of the layer.

Let us denote the number of reduced cylindrical capillaries [3] in a given section, having a radius from r to $r+dr$, through dn_s then the surface porosity (clearance) is equal to:

$$\Pi_s = A_l = \int_{r_{\min}}^{r_{\max}} \pi r^2 \frac{dn_s}{dr} dr \,, \qquad (5)$$

where n_s - the number of capillaries per unit area of the cross section of the body.

If A_l does not depend from L, that is surface porosity is the same in any section of the body, which is equivalent to the equality of the surface and bulk porosity of the body ($\Pi_V = \Pi_S$), then you can write:

$$u = \frac{\rho_l}{\rho_s} A_l = \frac{\rho_l}{\rho_s} \int_{r_{\min}}^{r_{\max}} \pi r^2 \frac{dn_s}{dr} dr \,. \qquad (6)$$

For many, especially for those rotating with a period π, the surface porosity of the body changes along the coordinates of the body, therefore, along with the differential volumetric characteristic of pores $f_V(r)$ the differential surface characteristic of the pores is introduced $f_S(r)$, equal

$$f_s(r) = \frac{dA_l}{dr} = \pi r^2 \frac{dn_s}{dr} \,. \qquad (7)$$

As an approximation, one can take $\Pi_V = \Pi_S$. One of the main structural characteristics of a porous body is the capillary pressure curve $p_c = f(\omega)$. In finely dispersed systems and capillary-porous bodies, the capillary pressure curves, being irreversible, are smooth and continuous, since discontinuities (a combination of rheons and isons) are insignificant and practically unobservable (Fig. 1).

This occurs because the capacity of the system (the porous body) is large compared to the volume of the liquid redistributed in the region of the rheon.

On the curves of capillary pressure distinguish: minimum saturation ω_{li} (wetting phase volume corresponding to high capillary pressures), which is independent of further pressure increase; residual saturation ω_{l0}, zero pressure; initial drying curve R_0 (displacement of the wetting phase from 100% saturation to ω_{li}); absorption curve A (increasing saturation from ω_{l0} to ω_{li}); secondary dehumidification curve R.

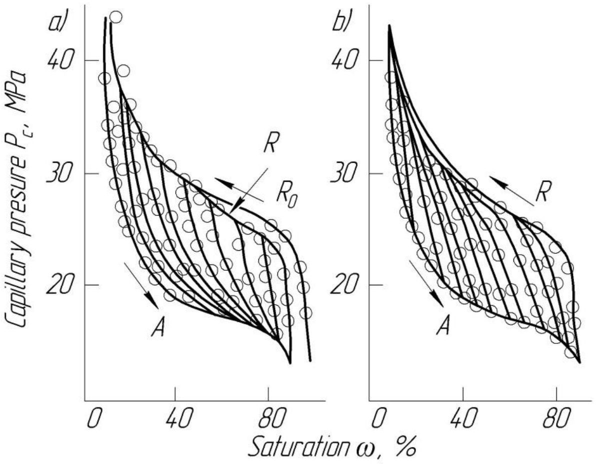

Figure 1 a, b – Capillary pressure hysteresis for microcapillaries with a diameter of 48 µm (a) and 60 µm (b)

Experimental data for many materials show that ω_{li} same for primary and secondary dehumidification curves.

Secondary drying and soaking curves form a hysteresis loop *RA* with all subsequent reference points lying inside this loop. Primary dehumidification sweep curves that start at the absorption curve *A*, either meet at the intersection of the curve (Fig. 1*b*) or converge on the secondary drying curve in the area close to the intersection. The behavior of the absorption sweep curves is similar. Any point within the hysteresis loop can be obtained in many ways.

The regularities of the capillary pressure curves presented above agree with the thermodynamic relations of the process of displacement of the wetting phase.

In the process of an intensive flow of liquid through a porous body, in addition to capillary and gravitational forces; Inertial forces are of great importance, which can be estimated by the Reynolds criterion (Re), Bond criteria (Bo) and Weber (We)

$$\mathrm{Re} = \frac{\rho \upsilon L}{\eta};\ Bo = \frac{\rho g L^2}{\sigma};\ We = \frac{\rho \upsilon^2 L}{\sigma}, \qquad (8)$$

where υ - fluid velocity; L - characteristic size.

The Reynolds criterion characterizes the ratio of inertial forces to viscous forces, the Bond criterion - the ratio of gravitational forces to capillary forces, and the Weber criterion - the ratio of inertial forces to capillary forces.

Combining the criterion We and Bo, one can obtain the well-known Froude criterion:

$$Fr = \frac{\upsilon}{\sqrt{gL}}, \qquad (9)$$

which characterizes the ratio of the forces of inertia to the forces of gravity.

There are modes when the dominant ones are: 1) inertia forces; 2) capillary forces; 3) gravitational. For small numbers We and Bo, the motion of the liquid is controlled by capillary forces (Fig. 2) and the effect of gravity can be neglected.

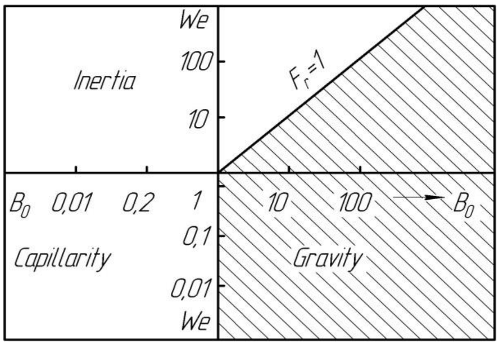

Figure 2 – Schematic relation of inertial, gravitational and capillary forces during fluid flow through a porous body

For small Bo numbers, the We criterion determines the decisive role of capillary and gravitational forces. For large values of the criterion Fr, the effects of gravity can also be neglected.

In table shows the molecular weight of materials for nanoparticles, on which the forces acting on rotating volumes of nanofluid in the pulley chamber.

Table – Molecular weight of nanomaterials for nanoparticles

Nanoparticle materials					
SiC	Zn	Cu	ZnO	Al	Al_2O_3
Molecular weight					
40,1	65,4	69,55	81,41	227,03	502,06

Diffusion-sedimentation processes in rotating systems. Consider thermodynamic equilibrium in the pulley rim chamber, i.e., the case when all flows and thermodynamic forces are simultaneously equal to zero. This case is of practical importance in determining molecular weights from sedimentation. Forces acting per unit mass of a component in a system that rotates at angular velocity ω, are described by a dependence of the form

$$\begin{gathered}(1-\rho\vartheta_k)(\omega^2 r + 2[\vartheta\omega]) - (grad\mu_k)_{p,T} = 0 \\ (k = 1, 2, \ldots, n-1),\end{gathered} \qquad (10)$$

where: $\omega^2 r$ – centrifugal force (r – distance from axis of rotation); ρ – nanofluid density; $2[\vartheta\omega]$ - Coriolis force; $grad\mu_k$ - entropy source intensity.

For a closed container, which is the pulley chamber, the speed ϑ_k, and hence the mass velocity ϑ are equal to zero at equilibrium. Therefore, (10) can be written in the form

$$(1-\rho\vartheta_k)\omega^2 r-(grad\mu_k)_{p,\,T}=0 \quad (k=1,\,2,\ldots,\,n-1). \tag{11}$$

This relation makes it possible to describe the distribution of chemical components at sedimentary equilibrium. Consider first a binary mixture (n = 2). Then from (11) we obtain

$$(1-\rho\vartheta_1)\omega^2 r-(grad\mu_1)_{p,\,T}=0, \tag{12}$$

where the first factor can otherwise be written as i.e., expressed in terms of the difference in the partial specific volumes of the two components. The chemical potential gradient can be represented as a gradient of some parameter characterizing the mixture composition, for example n_1, c_1, N_1 or ρ_1 (their values will be presented later):

$$(grad\mu_1)_{\rho,\,T}=\left(\frac{\partial\mu_1}{\partial n_1}\right)_{p,\,T} grad\ n_1=\left(\frac{\partial\mu_1}{\partial c_1}\right)_{p,\,T} grad\,c_1 \tag{13}$$

etc.

Using these relations, we get

$$\frac{grad\ n_1}{n_1}=M_1(1-\rho\vartheta_1)\left(1+\frac{\partial\ln\ f_1}{\partial\ln\ n_1}\right)^{-1}\frac{\omega^2 r}{RT}. \tag{14}$$

$$\frac{grad\ c_1}{c_1}=M_1\left\{1-c_1\left(1-\frac{M_2}{M_1}\right)\right\} (1-\rho\vartheta_1)\left(1+\frac{\partial\ln\ f_1}{\partial\ln\ n_1}\right)^{-1}\frac{\omega^2 r}{RT}. \tag{15}$$

$$\frac{grad\ \rho_1}{\rho_1}=\frac{grad\ N_1}{N_1}=M_1(1-\rho\vartheta_1)(1-c_1\rho\vartheta_1) \frac{1}{n_2}\left(1+\frac{\partial\ln\ f_1}{\partial\ln\ n_1}\right)^{-1}\frac{\omega^2 r}{RT}. \tag{16}$$

From here you can calculate the molecular weight M_1, if all other quantities are measured.

For ideal mixtures, for which the activity coefficient f_1 = 1, there is no multiplier containing this value. Then relation (14) reduces to the well-known Svedberg equation [3] for determining the molecular weight M_1 expressions on the left side (14) – (16) often considered equal to each other. It is clear from the foregoing which approximation this procedure corresponds to. In practice we usually have M_2 << M_1. Usually one deals with molar dilute solutions, so that $n_2 \approx 1$. If the solution is diluted so strongly that $c_2 \approx 1$, then the right sides of (15) and (16) are reduced to the right side of (11).

For *n*-component mixtures, using molar concentrations n_1 as composition parameters, we obtain

$$grad\mu_j=\sum_{i=1}^{n-1}\frac{\partial\mu_j}{\partial n_i}grad\ n_i. \tag{17}$$

From the equilibrium condition (10), taking into account (17), we find (*n* - 1) equations from which one can calculate (*n* - 1) quantities n_i, determining the distribution of matter in the system. In this way, for an ideal mixture, when μ_i is described by formula (14), we find

$$\frac{grad\ n_j}{n_j}=M_j(1-\rho\vartheta_j)\frac{\omega^2 r}{RT}. \tag{18}$$

Using (11) - (13), we express the gradients of mass concentrations in terms of gradients of molar concentrations:

$$\frac{grad\ c_j}{c_j}=\frac{grad\ n_j}{n_j}-\sum_{m=1}^{(n-1)}c_m\frac{M_m-M_n}{M_m}\frac{grad\ n_m}{n_m}. \tag{19}$$

Substituting (18) here, we obtain the relation which

$$\frac{grad\ c_j}{c_j}=\left\{M_j(1-\rho\vartheta_j)-\sum_{m=1}^{n-1}c_m(M_m-M_n)(1-\rho\vartheta_m)\right\} \frac{\omega^2 r}{RT}, \tag{20}$$

determines the mass distribution of the ideal *n*-components of the mixture at sedimentation equilibrium.

Another method for determining molecular weights is to study the rate of sedimentation. Experimentally, in the chamber of the brake pulley, the sedimentation rate of the layer between the nanofluid and the pure liquid is measured. In this case, both fluxes and thermodynamic forces are nonzero. We have phenomenological equations with thermodynamic forces. In practice, the Coriolis force can be neglected; then for an isotropic system the tensors L_{ik}^a reduced to scalars L_{ik}^a and the phenomenological equations take the form

$$J_i^a = \sum_{k,\ j=1}^{n-1} L_{ik}^a A_{kj}^a$$

$$\left\{(1-\rho\vartheta_j)\omega^2 r - \sum_{m=1}^{n-1}\mu_{jm}^x grad\ \ x_m\right\} \quad (21)$$

$$\frac{1}{T}\quad (i=1,\ 2,\ \ldots,\ n-1).$$

Introducing instead of the phenomenological coefficients L_{ik}^a diffusion coefficients D_{ik}^{bx}, we arrive at the following form of phenomenological equations:

$$J_i^a = \sum_{k,\ j=1}^{n-1} B_{ik}^{ab} D_{kj}^{bx}$$

$$\left\{\sum_{m=1}^{n-1}(\mu^x)_{jm}^{-1}(1-\rho\vartheta_m)\omega^2 r - grad\ \ x_j\right\} \quad (22)$$

$$(i=1,\ 2,\ \ldots,\ n-1).$$

For the case of binary mixtures (n = 2), this equation takes the form

$$J_1^a = B_{11}^{ab} D^{bx}\left\{(\mu^x)^{-1}(1-\rho\vartheta_1)\omega^2 r - grad\ \ x_1\right\}. \quad (23)$$

Using (23) and choosing $x = \rho_1$ and $b_2 = \rho\vartheta_2$, we get

$$J_i^a = \frac{a_2}{\rho_2\vartheta_2} D\left\{(\mu^\rho)^{-1}(1-\rho\vartheta_2)\omega^2 r - grad\ \rho_2\right\}. \quad (24)$$

where D – diffusion coefficient. In a state far from equilibrium, the system is homogeneous in a large region between the boundary (separating solution and pure solvent) and the outer wall, i.e., we have $grad\ \rho_1 = 0$. Therefore, in this region (24) we write in this way:

$$J_1^a = \frac{a_2}{\rho_2\vartheta_2} D(\mu^\rho)^{-1}(1-\rho\vartheta_1)\omega^2 r. \quad (25)$$

With the help of dependencies (4-7), this ratio can be rewritten in the following form:

$$M_1 = \frac{|\vartheta_1 - \vartheta^a|}{\omega^2 r}\ \frac{RTn_2}{D(1-\rho\vartheta_1)a_2}\left(1+\frac{\partial \ln\ f_1}{\partial \ln n_1}\right); \quad (26)$$

this shows what quantities you need to know in order to calculate the molecular weight M_1. In particular, it is necessary to measure the first factor, which is sometimes called the sedimentation factor.

Useful results are obtained with two specific ways of choosing weights a_i and the corresponding frame speed ϑ^a. In the first case, as ϑ^a choose the average molar velocity ϑ^m. At the same time, the weight a_2 equals n_2. Expression (26) then takes the form

$$M_1 = \frac{|\vartheta_1 - \vartheta^m|}{\omega^2 r}\ \frac{RT}{D(1-\rho\vartheta_1)}\left(1+\frac{\partial \ln\ f_1}{\partial \ln n_1}\right), \quad (27)$$

where R – gas constant; T – bulk temperature of the nanofluid.

This choice is convenient for molar diluted systems, since for such systems ϑ^m goes to zero. The second choice of practical interest is that we take $a_2 = \rho_2\vartheta_2$; it means, that ϑ^a is the average volumetric velocity. In this case, excluding ϑ_2 using the ratio $\rho_1\vartheta_1 + \rho_2\vartheta_2 = 1$, we find that (26) can be written in the form

$$M_1 = -\frac{\left|\vartheta_1 - \vartheta^o\right|}{\omega^2 r} \frac{RTn_2}{D(1-\rho\vartheta_1)(1-c_1\rho\vartheta_1)}\left(1+\frac{\partial \ln f_1}{\partial \ln n_1}\right). \tag{28}$$

It has been shown that the average volumetric velocity ϑ^o can be neglected in a number of important cases, in particular, if the system is a liquid enclosed in some vessel.

Just as it was done after relations (14) – (16), we can assume $n_2 \approx 1$ or (if, as usual in experiments in the pulley rim chamber, ($M_1 >> M_2$) $c_2 \approx 1$, which is a tougher assumption. Then from (22) and (23) we obtain in the case of an ideal nanofluid ($f_1 = 1$) Svedberg's famous expression:

$$M_1 = \frac{|\vartheta_1|}{\omega^2 r}\,\frac{RT}{D(1-\rho\vartheta_1)}. \tag{29}$$

Equations (28) and (29) are valid in the region where one can neglect grad c_1. Practically $|\vartheta_1|$ measured by observing part of the boundary layer, the motion of which is also determined by equation (28) and (29).

Thus, we deduced how the Svedberg equation corresponds to the sedimentation velocity, and the limits of their applicability were determined. In particular, all the quantities for the nonequilibrium state are determined just as rigorously as for the case of the equilibrium state.

The process of sedimentation in multicomponent mixtures can be investigated using equation (22) [4 - 7]. As a scale a_k and b_k again it is convenient to choose $\rho_k \vartheta_k$. Then the speed ϑ^a represents the average volumetric velocity, which, as discussed above, can often be taken equal to zero. At the same time, using as parameters x_j density ρ_j for regions, where gradients ρ_j can be neglected, we obtain instead of (22)

$$\rho_i \vartheta_i = \sum_{j=1}^{n-1} D_{ij}\left\{\sum_{m=1}^{n-1}\left(\mu^{\rho}\right)^{-1}_{jm}(1-\rho\vartheta_m)\,\omega^2 r\right\} \tag{30}$$
$$(i = 1,\ 2,\ \dots,\ n-1).$$

These equations make it possible to calculate molecular weights (similarly to the case of binary mixtures), if the necessary quantities included in the corresponding formulas are measured.

It has been shown above that diffusion coefficients can be used instead of phenomenological coefficients. Often, however, especially when considering systems that are in the field of external forces, completely different quantities, the so-called mobilities, are used instead of phenomenological coefficients. Below we show how the mobilities can be determined in the general case and how these quantities are related to the diffusion coefficients [8, 9].

Phenomenological equations have the form (24). Instead of transforming these equations to the form (25), we can write

$$J_1^a = \rho_i M_i \sum_{j=1}^{n-1} U_{ij}^a (1-\vartheta_j)\,\omega^2 r - \\ - \sum_{j,\ m=1}^{n-1} B_{ij}^{ab} D_{jm}^{bx}\, grad\ \ x_m \quad (i = 1,\ 2,\ \dots,\ n-1) \tag{31}$$

where U_{ij}^a - are called nanoparticle mobility. They have units of speed per unit of force per mole (or N molecules). A comparison of this equation with (31) shows that the mobilities are related to the phenomenological coefficients as follows:

$$\frac{1}{T}\sum_{k=1}^{n-1} L_{ik}^a A_{kj}^a = \rho_i M_i U_{ij}^a \tag{32}$$
$$(i,\ \ j = 1,\ 2, \dots,\ n-1).$$

When changing the main weights a_k the quantities on the right side of (32) are transformed in the same way as the flow J^a or diffusion coefficients D^{ax}. It should be noted that the mobilities are determined with respect to "arbitrary" velocities of reference systems ϑ^a, included in J_i^a, whereas the diffusion coefficients are defined relative to the "basic" velocity ϑ^b, whose choice is fixed. Eliminating the phenomenological coefficients from (32) [or comparing (31) with (32)], it is possible to establish a relationship between the mobilities of nanoparticles and diffusion coefficients:

$$\rho_i M_i U_{ij}^a = \sum_{k,\,m}^{n-1} B_{ik}^{ab} D_{km}^{bx} \left(\mu^x\right)_{mj}^{-1} \quad (i,\ j = 1, 2, \ldots, n-1). \tag{33}$$

This relationship is the generalized Fokker-Einstein relation. If you chose $b_k = \vartheta_k \rho_k$ and $x_k = \rho_k$, as in the previous cases, then relation (33) would take the form

$$\rho_i M_i U_{ij}^a = \sum_{k,\,m}^{n-1} B_{ik}^{a0} D_{km} \left(\mu^\rho\right)_{mj}^{-1} \quad (i,\ j = 1, 2, \ldots, n-1). \tag{34}$$

For binary mixtures (n = 2) we have $B_{11}^{a0} = a_2 / \rho_2 \vartheta_2$. hen the relation between the mobility U_{11}^a, which we denote simply by U^a, and the diffusion coefficient takes the form

$$\rho_1 M_1 U^a = \frac{a_2}{\rho_2 \vartheta_2} D\left(\mu^\rho\right)^{-1}. \tag{35}$$

Taking into account [4-7], this relation can be written as:

$$D = \frac{n_2}{a_2} RTU^a \left(1 + \frac{\partial \ln f_1}{\partial \ln n_1}\right). \tag{36}$$

For an ideal system, the activity coefficient $f_1 = 1$, so the resulting relation simplifies:

$$D = \frac{n_2}{a_2} RTU^a. \tag{37}$$

Finally, if we choose the “molar” description, when the weight factor $a_2 = n_2$, then we get

$$D = RTU^m, \tag{38}$$

i.e., the Fokker-Einstein relation.

We proceed to discuss the results obtained.

The discussion of the results

Theoretical and experimental studies of non-uniform nanocapillary and nanofluid cooling of friction pairs of a model tape-shoe brake of a drawworks made it possible to state the following:

- a relationship has been established between sedimentation and diffusion in a nanofluid due to the intensity of the entropy source: sedimentation is assessed using the gradient theory when determining the sedimentation coefficient; diffusion of nanoparticles was characterized by the coefficient and their mobility in the liquid; the selection by molecular weight of various materials of nanoparticles was made;
- means providing local heat removal are briquettes made of nanoparticles from various materials and forming capillary structures resting in metal perforated frames; the latter are installed in three rows of different widths (circular grooves on the inner surface of the pulley rim, to which the fluid chamber is attached);
- it has been established that the thermal conductivity of nanoparticles in briquettes with different capillary structures does not go beyond a certain limit level with an increase in their concentration; the thermal conductivity coefficient of nanoparticles in briquettes is

some integral property of this non-standard two-phase system.

Conclusion

Thus, the workability of nanoparticles in capillary structures in rotating brake systems has been justified.

Conflict of Interests

The authors declare there is no conflict of interests related to the publication of this article.

REFERENCES

1. Tribology: friction, wear, lubrication / A.Kh. Janahmadov, A.I. Volchenko, K.T. Nabizade [et al.] // Baku: *Apostrophe-A*, 2019. 640 p. *(in Russian)*

2. Design and verification calculation of friction units of band-shoe brakes of drilling winches: Standard: A.Kh. Janahmadov, D.A. Volchenko, N.A. Volchenko [and others] // Baku: *Apostrophe.* – 311 p. *(in Russian)*

3. Reference book on tribotechnics. In 3 vols. T. 1. Theoretical foundations / ed. ed. M. Hebdy, A.V. Chichinadze. - Moscow: *Mashinostroenie*, 1989. - 400 p. *(in Russian)*

4. Smirnov L.I., Goltsov V.A. Dynamics of hydrogen concentration inhomogeneities in metals // *Physics of Metals and Metal Science.* - 1998. - T. 85, no. 2. - S. 19 - 23. *(in Russian)*

5. Smirnov L.I., Goltsov V.A. Equation of diffusion of interstitial atoms in metals at arbitrary concentration gradients // *Physics of Metals and Metal Science.* - 1998. - T. 85, No. 5. - Pp. 44 - 47. *(in Russian)*

6. Barlag H., Opara L., Zuchner H. Hydrogen diffusion in palladium based f.c.c. alloys // J.A. Alloys and Compounds. - 2002. - Vol. 330 - 332. - Pp. 434 - 437. *(in English)*

7. Progress in Hydrogen Treatment of Materials / Aoki K., Artemenko Yu. A., Borisov G. P. et al. Ed. V. A. Golstov. - Donetsk - Coral Gables: *Kassiopeya*, 2001. - 544 p. *(in Russian)*

8. Volchenko N.A. Electrochemistry with double electrical layers in frictional interaction metal-polymer tribolink / Volchenko N.A., Krasin P.S., Volchenko D.A., Voznyi A.V. // IOP Conference Series and Engineering, 2018, 327(3), 032059. *(in English)*

9. Volchenko, N.A. Pulse-contact frictional interaction of microprotrusions of friction pairs of brake devices / N.A. Volchenko, D.A. Volchenko, P.A., Polyakov, E.S Fedotov, A.S. Evchenko // IOP Conference Series: Materials Science and Engineering, 2019, 560 (1), 012194. *(in English)*