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## **Development of a Mathematical Model and Solution to the Problem of Optimizing the Vacuum Block of Oil Distillation Unit**

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### **Abstract**

As it is known, one of the initial and important stages in the creation of optimal control systems of oil refining technological units is the development of a mathematical model that can adequately record the processes at any time. Studies have shown that the lack of information about the state of complex oil refining processes in many cases reduces their efficiency and effectiveness. On the other hand, the wide range of both quality and quantity of raw materials for processing makes their efficiency even more unsatisfactory. Under these conditions, it is difficult to develop mathematical models that can adequately describe the static modes of technological processes. Due to this the development of mathematical models is relevant both in scientific and practical terms. Taking into account the above, in the presented paper is developed a deterministic mathematical model that can adequately reflect the current technological operating conditions of a vacuum unit, which is the main device of the vacuum column of the primary oil refining technological complex. On the basis of this model, an optimization algorithm is developed that allows to calculate the optimal operating modes of the vacuum block.

**Keywords:** oil refinery, K-4 column, mathematical model, adequacy, optimization.

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## **Neftin distillə olunması qurğusunun vakuum blokunun riyazi modelinin işlənilməsi və optimallaşdırılması məsələsinin həlli**

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### **Xülasə**

Tədqiqatlar göstərmişdir ki, mürəkkəb neft emalı proseslərinin vəziyyəti haqqında məlumatın tam olmaması bir çox hallarda onların səmərəliliyini azaldır. Digər tərəfdən, qurğuya emal üçün verilən xammalın həm keyfiyyət, həm də kəmiyyət baxımından geniş diapozonda dəyişməsi onun idarə olunmasının keyfiyyətini qeyri-qənaətbəxş edir. Belə bir şəraitdə texnoloji proseslərin statik rejimlərini adekvat təsvir edə bilən riyazi modellərinin işlənilməsi çətinləşir. Bu baxımdan riyazi modellərin işlənilməsi həm elmi, həm də praktiki baxımdan aktualdır. Yuxarıda göstərilənləri nəzərə alaraq təqdim olunan məqalədə neftin emalı texnoloji kompleksinin əsas qurğusu olan vakuum kalonunun cari texnoloji iş rejimini adekvat şəkildə əks etdirə bilən determinə olunmuş riyazi model işlənilib hazırlanmışdır. Bu model əsasında vakuum blokunun optimal iş rejimlərini hesablamağa imkan verən optimallaşdırma alqoritmi işlənilmişdir.

**Açar sözlər:** neft emalı qurğusu, K-4 kolonu, riyazi model, adekvatlıq, optimallaşdırma.

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## **Разработка математической модели и решение задачи оптимизации вакуумной установки установки переработки нефти**

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### **Аннотация**

Исследования показали, что отсутствие информации о состоянии сложных процессов нефтепереработки во многих случаях снижает их эффективность и результативность. С другой стороны, широкий диапазон как по качеству, так и по количеству сырья для переработки делает их эффективность еще более неудовлетворительной. В этих условиях сложно разработать математические модели, способные адекватно описывать статические режимы технологических процессов. В связи с этим разработка математических моделей актуальна как в научном, так и в практическом плане. Учитывая вышеизложенное, в представленной работе разработана детерминированная математическая модель, способная адекватно отражать текущие технологические условия работы вакуумной колонны, которая является основным устройством технологического комплекса первичной переработки нефти. На основе данной модели разработан алгоритм оптимизации, позволяющий рассчитать оптимальные режимы работы вакуумного блока.

**Ключевые слова:** нефтеперерабатывающая установка, колонна К-4, математическая модель, адекватность, оптимизация.

## **Introduction**

It is known that modern technological complexes of oil refining as control objects, as a rule, have a large number of controlled and uncontrolled parameters, a lack of information in terms of providing information characterizing the progress of the process, high production capacity, the presence of mutual connections between technological devices, the various technological devices of these devices differ a set of connections and continuity of technological operations [1, 2].

The studies have shown that the main devices of the vacuum block of the technological complex are: a tubular furnace that heats fuel oil; vacuum column for fuel oil rectification; additional evaporator columns.

It should be noted that the main device of this unit in terms of producing targeted products as a control object is the K-4 column, which carries out the rectification of fuel oil under vacuum conditions. The main technological mode parameters characterizing this apparatus are temperatures and residual pressure at different points - at the top and bottom of the column. In order to maintain and control the temperatures at different points in column, the irrigation given to those points is used. It is especially important to keep the temperature constant at the bottom of the column. Thus, its change manifests itself in the heat balance of fuel oil vapors rising up along the column, which causes the creation of additional exciting factors. Temperature control in the plates receiving heavy and light oil fractions in the column (at the inlet of additional evaporator columns has a greater effect on the yield and quality indicators of the obtained products.

As a result of the study of this apparatus as a control object, it was found that changes

in the consumption and quality indicators of raw materials - fuel oil, removing from the atmospheric block for processing, manifest themselves as a more disturbance factor. In addition to these factors, other disturbance effects in the system can be compensated for.

An analysis of the scientific literature confirms that any technological complex, including an oil distillation unit, can be studied as a deterministic or incomplete system, depending on the amount of a priori information characterizing its current technological state. Experience has shown that in most cases, one of the main reasons for the ineffectiveness of such systems is related to the inadequacy and non-universality of their mathematical models. In this regard, when operating the optimal control system of the vacuum block of the oil distillation technological complex, the main focus is the building of a complex of more adequate mathematical models.

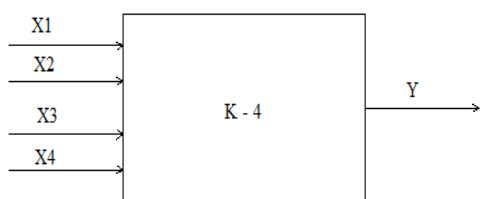
## **Purpose of the work**

Development of a deterministic mathematical model that can adequately describe the current technological conditions of the main apparatus of the vacuum block of the oil distillation unit and the solution of the problem of optimization for the static modes of the unit on the basis of this model.

## **Problem statement**

The mathematical formulation of the modeling problem, which expresses the regression dependence between light oil fraction yield of the K-4 column of vacuum section and other parameters, and the sequence of solving the problem according to the formulation are as follows. The diagram of the structural form of K-4 column as a

modeling facility for the building of a mathematical model is depicted in Fig. 1.



**Figure 1** – Structural diagram of K-4 column as a modeling facility

Where,  $X_1$  — fuel oil consumption in the K-4 column,  $m^3/h$ ;  $X_2$  — temperature of fuel oil in the K-4 column,  $^{\circ}C$ ;  $X_3$  — level in the K-4 column, %;  $X_4$  — pressure at the top of the K-4 column,  $kq/sm^2$ ;  $Y$  — yield of light oil fraction,  $m^3/h$ .

Constraint conditions set for input and control parameters.

The linear mathematical model of K-2 column is written as follows (with unknown coefficients):

$$y = b_0 + \sum_{i=1}^n b_i x_i.$$

here  $b_i$  — unknown regression coefficients.  $x_i$  is input and  $y_i$  is output.

There are many ways to find the unknown regression coefficients of a mathematical model, one of which is to find the regression coefficients using the least squares method [3, 4]. This method is based on the fact that the square of the difference between the calculated value and the experimental one is very small. The initial statistical data obtained from the passive experiments required for modeling are as shown in table 1.

**Table 1** – Statistical data table

$X_1$	$X_2$	$X_3$	$X_4$	$Y$
40.15	172.77	29.5	58.3	25.19
41.23	174.5	29.6	58.9	24.87
41.56	175	29.8	58.6	24.2
40.39	177	29.1	58.96	24.2
42.69	183.2	29.1	58.32	24.87
42.4	185.1	29.7	59.32	24.2
42.63	172.7	29.6	59.62	25.2
42.5	172.3	29.3	60.34	24.2
43.6	172.6	29.3	60.4	24.2
43.56	173.1	29.2	60.44	24.87
43.65	172.9	30.1	60.55	24.87
43.89	183.5	30.2	61.2	25.18
41.23	185.2	30.5	62.61	24.2
42.56	182.2	30.5	61.87	24.2
40.89	171.1	30.7	61.3	24.2
40.1	174.56	30.1	61.15	24.2
40	177.8	30	59.65	24.2
41.2	179.5	29.8	60.25	24.2
41.23	176.9	30.7	60.36	24.2
41.65	185.7	30.8	61.97	24.45
41.66	183.8	30.54	58.67	24.45
41.65	180.02	30.12	59.45	24.2
41.32	179.6	30.46	59.46	24.17
41.9	178.2	30.66	59.2	24.19

## Solving of the problem

### Development of the mathematical model

The Matlab program is used to build the mathematical model of the object [5, 6].

$$Y = B_0 + B_1X_1 + B_2X_2 + B_3X_3 + B_4X_4$$

We enter the data from experiments into the program.

The values of fuel consumption:

$X_1 = [40.15 \ 41.23 \ 41.56 \ 40.39 \ 42.69 \ 42.4 \ 42.63 \ 42.5 \ 43.6 \ 43.56 \ 43.65 \ 43.89 \ 41.23 \ 42.56 \ 40.89 \ 40.1 \ 40 \ 41.2 \ 41.23 \ 41.65 \ 41.66 \ 41.65 \ 41.32 \ 41.9];$

The values of temperature:

$X_2 = [172.77 \ 174.5 \ 175 \ 177 \ 183.2 \ 185.1 \ 172.7 \ 172.3 \ 172.6 \ 173.1 \ 172.9 \ 183.5 \ 185.2 \ 182.2 \ 171.1 \ 174.56 \ 177.8 \ 179.5 \ 176.9 \ 185.7 \ 183.8 \ 180.2 \ 179.6 \ 178.2];$

The values of level:

$X_3 = [29.5 \ 29.6 \ 29.8 \ 29.1 \ 29.1 \ 29.7 \ 29.6 \ 29.3 \ 29.3 \ 29.2 \ 30.1 \ 30.2 \ 30.5 \ 30.5 \ 30.7 \ 30.1 \ 30 \ 29.8 \ 30.7 \ 30.8 \ 30.54 \ 30.12 \ 30.46 \ 30.66];$

The values of pressure:

$X_4 = [58.3 \ 58.9 \ 58.6 \ 58.96 \ 58.32 \ 59.32 \ 59.62 \ 60.34 \ 60.4 \ 60.44 \ 60.55 \ 61.2 \ 62 \ 61.87 \ 61.3 \ 61.15 \ 59.65 \ 60.25 \ 60.36 \ 61.97 \ 58.67 \ 59.45 \ 59.46 \ 59.2];$

The values of yield of light oil fraction:

$Y = [25.19 \ 24.87 \ 24.2 \ 34.2 \ 34.87 \ 34.2 \ 35.2 \ 34.2 \ 34.2 \ 34.87 \ 34.87 \ 34.2 \ 34.2 \ 34.2 \ 34.2 \ 34.2 \ 34.2 \ 34.2 \ 34.2 \ 34.45 \ 34.45 \ 34.2 \ 34.17 \ 34.19];$

The number of experiments:

$$K=24$$

As a result the program calculates the coefficients of the equation:

$$B_0 = 61.4590, B_1 = 1.1740, B_2 = 0.2003, B_3 = 0.2003, B_4 = -1.1603$$

The regression equation in a linear form is formulated as follows:

$$Y = 61.4590 + 1.1740X_1 + 0.2003X_2 - 0.8598X_3 - 1.1603X_4 \quad (1)$$

In order to check the adequacy of the developed linear mathematical model, its analysis should be carried out, which can also be performed using Excel. The results of the analysis include residual error, set correlation coefficient, Fisher criterion are shown in table 2.

**Table 2** – Adequacy indicators of the mathematical model

The values obtained from the analysis	
Fisher criterion	3.9375
R square	0.172415
Residual error	0.316329
Set correlation coefficient (R)	0.941523
The number of experiments	24

The Fisher criterion was used to check the adequacy of the mathematical model.

The experimental value of Fisher criterion ( $F_{exp}$ ) must be greater than its table value ( $F_{tab}$ ) for the mathematical model to be adequate:

$$F_{exp} < F_{tab}. \quad (2)$$

When checking the adequacy according to set correlation coefficient, the closer the coefficient R is to the unit ( $0 \leq R \leq 1$ ), the more adequate the mathematical model is.

### Solving of the optimization problem

For solving of optimization problem it is used the mathematical program MathCad taking into account the objective function and the limitation to input and control effects [7,8]. Based on the statistical obtained data, the limitations will be as follows:

x

$$b_0 := 61.4590$$

$$b_1 := 1.1740$$

$$b_2 := 0.2003$$

$$b_3 := -0.8598$$

$$b_4 := -1.1603$$

$$f(x_1, x_2, x_3, x_4) := b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + b_3 \cdot x_3 + b_4 \cdot x_4$$

$$x_1 := 43.56$$

$$x_2 := 173.1$$

$$x_3 := 29.2$$

$$x_4 := 60.44$$

Given

$$40.1 \leq x_1 \leq 44$$

$$171.1 \leq x_2 \leq 185.7$$

$$29.1 \leq x_3 \leq 30.96$$

$$58.3 \leq x_4 \leq 61.97$$

$$p := \text{Maximize}(f, x_1, x_2, x_3, x_4)$$

$$p = \begin{pmatrix} 44 \\ 185.7 \\ 29.1 \\ 58.3 \end{pmatrix} \blacksquare$$

$$f(x_1, x_2, x_3, x_4) = 25.18$$

Optimal value of objective function:

$$F_{\text{opt}}=25.18$$

Where,  $f(x_1, x_2, x_3, x_4)$  – is function characterizing the yield of the light oil fraction depends on values of  $x_1$ - fuel oil consumption in the K-4 column,  $x_2$ - temperature of fuel oil in the K-4 column,  $x_3$ -level in the K-4 column.  $x_4$  - pressure at the top of the K-4 column According to the conditions of the problem, the objective function should be maximum. As can be seen, the maximum value of the objective function is 25.18.

Table 3 shows a comparison of actual and optimal values of parameters.

**Table 3** – Results of optimal solution for reaction products

Parameters	Actual value	Optimal value
fuel oil consumption in the K-4 column,	43.56	44
temperature of fuel oil in the K-4 column	173.1	185.7
level in the K-4 column	29.2	29.1
pressure at the top of the K-4 column	60.44	58.3
yield of light oil fraction	24.87	25.18

## Conclusion

In the presented paper, a deterministic mathematical model has been developed that

can adequately describe the current technological operating conditions of the main apparatus of the vacuum block of the oil distillation technological complex and the adequacy of the model has been checked and confirmed. On the basis of this model, the problem of optimization, which allows determining the optimal operating mode of the vacuum column is solved and the optimal values of the main regime parameters of the vacuum column are calculated.

## Conflict of interests

The author declares there is no conflict of interests related to the publication of this article.

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