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Construction of the "Moment-Curvature" Scheme for Annular Cross-Sectional Reinforced Concrete Elements and its Application in the Calculatıon of Reinforced Concrete Beams

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Abstract

In solving the issues of rigidity of reinforced concrete elements, the determination of the displacements of these elements plays an important role. As you know, in order for these displacements to be determined, there must be a “moment-curvature" diagram. The authors have developed an effective numerical method for constructing a tabular dependence of the moment curvature diagram. In the article, an analytical expression was proposed that approximates diagrams in the form of a table with high accuracy, and the application of this dependence to the determination of deflections of reinforced concrete beams is shown. Based on the proposed methodology, displacements at an arbitrary load level can be determined using a single algorithm. It is shown that the influence of nonlinear deformation diagrams of materials on the value of deflections is strong, whereas the influence of static unsolvable diagrams on static unsolvability is weak. The effectiveness of the proposed calculation method was demonstrated by numerical examples.

Keywords: bending moment, curvature, reinforcement, concrete, deformation, stress, deflection, static unsolvable beam.

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Həlqəvi en kəsikli dəmirbeton elementlər üçün "moment-əyrilik" diaqramının qurulması və onun dəmirbeton tirlərin heasblanmasına tətbiqi

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Xülasə

Dəmirbeton elementlərin sərtlik məsələlərinin həllində bu elementlərin yerdəyişmələrinin təyini önəmli rol oynayır. Məlum olduğu kimi həmin yerdəyişmələrin təyin oluna bilməsi üçün "moment-əyrilik" diaqramı mövcud olmalıdır. Müəlliflər tərəfindən moment əyrilik diaqramının cədvəl formalı asılılığının qurulması üçün effektiv ədədi metodika işlənmişdir. Məqalədə cədvəl şəkilli diaqramları yüksək dəqiqliklə approksimə edən analitik ifadə təklif olunmuş və həmib asılılığın dəmirbeton tirlərin əyintilərinin təyininə tətbiqi göstərilmişdir. Təklif olunmuş metodika əsasında yükləmənin ixtiyari səviyyəsində vahid alqoritm əsasında yerdəyişmələr təyin oluna bilir. Göstərilmişdir ki, materialların qeyri xətti deformasiya diaqramlarının əyintilərin qiymətinə təsiri güclü olduğu halda statik həll olunmayan tirlərin statik həll olunmazlığına təsiri zəifdir. Təklif olunmuş hesablama metodikasının effektivliyi ədədi misallarla nümayiş olunmuşdur.

Açar sözlər: moment, əyrilik, armatur, beton, deformasiya, gərginlik, əyinti, statik həll olunmayan tir.

Построение диаграммы «момент-кривизна» для железобетонных элементов кольцевого сечения и ее применение к расчету железобетонных балок

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Аннотация

При решении задач жесткости железобетонных элементов важную роль играет определение перемещений этих элементов. Как известно, для определения премещений важно наличие диаграммы «момент – кривизна». Авторами разработана эффективная численная методика для построения табличной формы данной диаграммы. В статье предложены аналитические зависимости, аппроксиммующие табличные формы диаграммы «момент – кривизна» и показано применение этих зависимостей для определения перемещений железобетонных балок. Предложенная методика позволяет для любого уровня загружения по единому алгоритму определить прогибы железобетонных балок. Показано, что влияние нелинейности диаграммы деформирования материалов на прогибы балок очень сильно, но мало влияет на статическую неопределимость. Эффективность предложенной методики расчета демонстрирована на численных примерах.

Ключевые слова: момент, кривизна, арматура, бетон, деформация, напряжение, прогиб, статически неопределимая балка.

Introduction

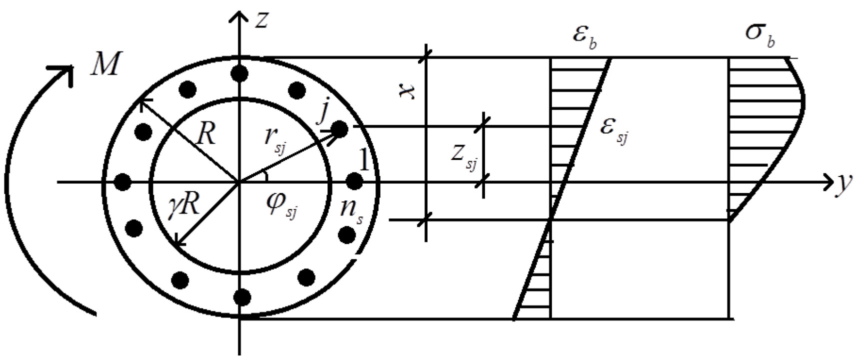
When studying the state of stress-strain deformation during bending of the annular cross-section of a reinforced concrete element, it is assumed that for an arbitrary load level, the plane section hypothesis for an inhomogeneous element turns out to be correct. This is one of the main provisions adopted when creating the theory of calculation of reinforced concrete structures based on a modern nonlinear deformation model [1,5,7,8]. The work of concrete in tension is not taken into account and it is believed that the tension stresses in the cross section are perceived by the reinforcement rods [6]. Well-known specialists in the theory of reinforced concrete structures believe that the state of stress-strain deformation and the load-bearing capacity of reinforced concrete structures as a whole are possible only with the use of realistic nonlinear diagrams of deformation of materials [2,3,4]. It is assumed that the reinforcing bars deform based on a symmetrical diagram with a limited yield area in tension and compression. And the work of concrete during compression is the fraction proposed in the Eurocode - expressed by a rational diagram [6,7]. The application of real non-linear deformation diagrams of materials generally allows to monitor the state of stress deformation formed in reinforced concrete structures depending on the level of loading. The construction of moment-curvature diagrams for reinforced concrete elements with a complex cross-sectional shape based on the nonlinear deformation model is one of the actual problems of the theory of modern construction structures and is of great practical importance.

Construction of "moment-curvature" diagram. Using the indicated diagrams of deformation of materials in scheme 1, the following expressions were obtained for the internal normal force and bending moment, which are formed as if due to compressive stresses in concrete:

$$N_b = 2 \cdot R^2 \cdot R_b \cdot N_b^*(\beta, \xi),$$
$$M_b = 2 \cdot R^3 \cdot R_b \cdot M_b^*(\beta, \xi) \qquad (1)$$

Where

$$f(\beta, \xi, \bar{z}) = \frac{k \cdot \dfrac{\beta}{\xi} \cdot (\xi - 1 + \bar{z}) - \left(\dfrac{\beta}{\xi}\right)^2 \cdot (\xi - 1 + \bar{z})^2}{1 + (k-2) \cdot \dfrac{\beta}{\xi} \cdot (\xi - 1 + \bar{z})}$$



Scheme 1 – Calculation scheme of a reinforced concrete element of an annular cross-section works in bending

$$Y_{Nb}(\beta, \xi, a) = \int_a^b f(\beta, \xi, \bar{z}) \cdot \sqrt{a^2 - \bar{z}^2} \cdot d\bar{z}, \quad Y_{Mb}(\beta, \xi, a, b) = \int_a^b f(\beta, \xi, \bar{z}) \cdot z \cdot \sqrt{a^2 - \bar{z}^2} \cdot d\bar{z}$$

$$N_b^*(\beta, \xi) = \begin{cases} Y_{Nb}(\beta, \xi, 1-\xi, 1) - Y_{Nb}(\beta, \xi, -\gamma, \gamma); & when \quad 1+\gamma \le \xi < 2 \\ Y_{Nb}(\beta, \xi, 1-\xi, 1) - Y_{Nb}(\beta, \xi, 1-\xi, \gamma); & when \quad 1-\gamma \le \xi < 1+\gamma \\ Y_{Nb}(\beta, \xi, 1-\xi, 1); & when \quad 0 < \xi < 1-\gamma \end{cases}$$

$$M_b^*(\beta, \xi) = \begin{cases} M_{Nb}(\beta, \xi, 1-\xi, 1) - M_{Nb}(\beta, \xi, -\gamma, \gamma); & when \quad 1+\gamma \le \xi < 2 \\ M_{Nb}(\beta, \xi, 1-\xi, 1) - M_{Nb}(\beta, \xi, 1-\xi, \gamma); & when \quad 1-\gamma \le \xi < 1+\gamma \\ M_{Nb}(\beta, \xi, 1-\xi, 1); & when \quad 0 < \xi < 1-\gamma \end{cases} \qquad (2)$$

Based on the accepted deformation schemes, the normal force and bending moment on the armature rods are determined as follows:

$$N_s^*(\beta,\xi) = \sum_{j=1}^{n_s} \sigma_{sj} \cdot A_{sj}; \qquad M_s^*(\beta,\xi) = \sum_{j=1}^{n_s} \sigma_{sj} \cdot A_{sj} \cdot r_{sj} \cdot \sin\varphi_{sj} \qquad (3)$$

Where

$$\sigma_{sj} = \begin{cases} E_{sj} \cdot \varepsilon_{sj}; & when \quad |\varepsilon_{sj}| = \left| \frac{\varepsilon_R \cdot \beta}{\xi} \cdot \left(\xi - 1 + \frac{r_{sj}}{R} \cdot \sin\varphi_{sj} \right) \right| \le \varepsilon_{sj,ax} \\ R_{sj} \cdot \frac{\varepsilon_{sj}}{|\varepsilon_{sj}|}; & when \quad |\varepsilon_{sj}| = \left| \frac{\varepsilon_R \cdot \beta}{\xi} \cdot \left(\xi - 1 + \frac{r_{sj}}{R} \cdot \sin\varphi_{sj} \right) \right| > \varepsilon_{sj,ax} \end{cases} \qquad (4)$$

In the above equations $\beta = \frac{\varepsilon_b}{\varepsilon_R}$ - the level of deformation on the compressible surface of the section, $\xi = \frac{x}{R}$ - a dimensionless parameter that determines the position of the neutral axis, -elastic modulus of the armature beams, E_{sj} - deformations of the armature rods corresponding to the beginning of the yield area, $\varepsilon_{sj,ax}$ - the distance of the center of gravity of the armature rods from the center of the cross section. Based on the expressions obtained, the equilibrium equations of the cross section for an arbitrary load level are written as follows:

$$2 \cdot R^2 \cdot R_b \cdot N_b^*(\beta,\zeta) + N_s^*(\beta,\zeta) = 0 \qquad (5)$$

$$2 \cdot R^3 \cdot R_b \cdot M_b^*(\beta,\zeta) + M_s^*(\beta,\zeta) = M \qquad (6)$$

Here M is the bending moment acting on the cross section. When constructing the "moment-curvature" diagram based on the equalities (5) and (6), the numerical technique proposed by the first author [8] is used. Its essence lies in the fact that, since the interval of change of β parameter is known in advance, a certain value is assigned to this parameter from the area of its change, and from equality (5) ξ parameter is determined as the root of a one-dimensional nonlinear equation with the required accuracy. Further, on the basis of equality (6) and the known values of β and ξ parameters, the value of the bending moment formed in the section corresponding to the accepted value of β parameter is calculated. After that, and based on the known values of β and ξ parameters, it can be calculated based on the equation of the curvature of the section

$$\chi = \frac{\beta \cdot \varepsilon_R}{\xi \cdot R} \tag{7}$$

and, thus, obtain solutions such as *(β, ξ, χ, M)* the solution of the equation of the system above. Consequently, for the considered element of the annular cross-section, increasing β parameter by a certain step in the area of its change, the ordinates of the "moment-curve" diagram are determined. The proposed solution algorithm allows you to set the desired parameters with any accuracy and is very easy to program. Such a programm module was construct, and various numerical experiments were carried out using it. Based on the conducted numerical experiments, moment-curvature diagrams were constructed and analyzed for various variants both in tabular and graphical form and the following dependence was proposed for the analytical approximation of these diagrams.

$$\chi = \frac{M}{B_0} \cdot \left[1 + \eta \cdot \left(\frac{M}{M_u} \right)^m \right] \tag{8}$$

To determine the three unknowns B_0, η and m the parameters included in this equality, provided that when $\chi = \chi_u$ then $M = M_u$, the values of the experimental dependence coincide at any point (M_0, χ_0) of the analytical approximation dependence and to determine them using the conditions of equality of the areas under the approximation and experimental curves, the following system equation is constructed

$$\begin{cases} \chi_u = \dfrac{M_u}{B_0} \cdot (1 + \eta) \\ \chi_0 = \dfrac{M_0}{B_0} \cdot \left[1 + \eta \cdot \left(\dfrac{M_0}{M_u} \right)^m \right] \\ \omega_0 = \dfrac{M_u^2}{B_0} \cdot \left(\dfrac{1}{2} + \dfrac{\eta}{m+2} \right) \end{cases} \tag{9}$$

Here $\omega_0 = \sum_{j=1}^{n} \frac{\chi_j + \chi_{j-1}}{2} \cdot \left(M_j - M_{j-1} \right)$ is the area of the experimental diagram obtained in the form of a table. System (9), designed to determine unknown parameters, is a system of nonlinear equations. We bring this system to the solution of a nonlinear equation taking into account one of the parameters. For known quantities $\gamma_1 = \frac{\chi_0}{\chi_u}$ and $\gamma_2 = \frac{M_0}{M_u}$ by dividing the first two equations of the system (9) fromside to side, by entering their notation, we express the parameter η with the parameter m

$$\eta = \frac{\gamma_1 - \gamma_2}{\gamma_2^{m+1} - \gamma_1} \tag{10}$$

Using this, the parameter B_0 can also be expressed through the parameter m as follows

$$B_0 = \frac{M_u}{\chi_u} \cdot \left(1 + \frac{\gamma_1 - \gamma_2}{\gamma_2^{m+1} - \gamma_1} \right) \tag{11}$$

Finally, given the equalities (10) and (11) in the third equality of the system (9), we obtain the following unambiguous nonlinear equation to determine the parameter m:

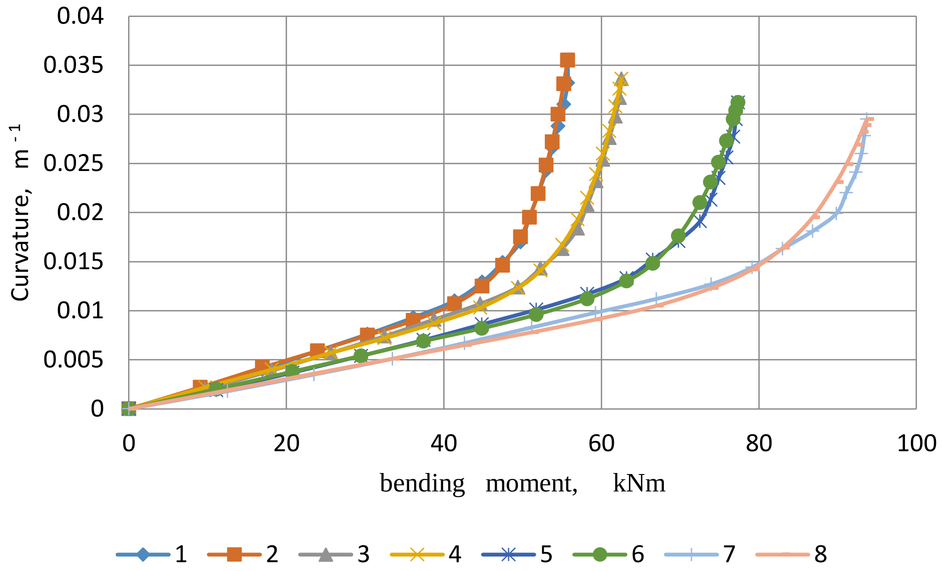
$$\frac{M_u \cdot \chi_u \cdot \left(\gamma_2^{m+1} - \gamma_1 \right)}{\gamma_2^{m+1} - \gamma_2} \cdot \left(\frac{1}{2} + \frac{1}{m+2} \cdot \frac{\gamma_1 - \gamma_2}{\gamma_2^{m+1} - \gamma_1} \right) - \omega_0 = 0 \tag{12}$$

After this equation is solved by known numerical methods, such as the method of dividing the workpiece in half and m parameter is determined, the remaining two parameters can also be easily calculated based on the above equalities. The programm module that implements the definition of the desired parameters was compiled and calculations were performed with its use in the presence of concrete $B\,20$, $R_b = 11{,}5\ MPa$, geometric cross-section dimensions $R = 15\ sm$, $r = 8\ sm$ thickness of the protective layer $a_1 = 2\ sm$, reinforcement *A 400*, $R_s = 350\ MPa$, and their results are shown in Table 1 below.

Table 1 – Ordinates of the "moment-curvature" diagram for various reinforcement schemes

$12\varnothing12$			$12\varnothing14$		
M, kN	χ, m^{-1}	χ, m^{-1} on the basis of equality (8) $B_0 = 4060{,}39\ kN\cdot m^2$ $\eta = 1{,}5875$; $m = 11{,}5061$	M, kN	χ, m^{-1}	χ, m^{-1} on the basis of equality (8) $B_0 = 4501{,}20\ kN\cdot m^2$ $\eta = 1{,}4178$; $m = 10{,}6097$
0	0	0	0	0	0
9,077	0,0021	0,0022	9,706	0,0020	0,0022
16,951	0,0040	0,0042	18,138	0,0039	0,0040
23,954	0,0059	0,0059	25,654	0,0057	0,0057
30,297	0,0076	0,0075	32,480	0,0074	0,0072
36,121	0,0093	0,0090	38,768	0,0091	0,0087
41,349	0,0110	0,0107	44,627	0,0107	0,0103
44,852	0,0129	0,0125	49,412	0,0124	0,0123
47,461	0,0149	0,0146	52,263	0,0143	0,0141
49,726	0,0170	0,0175	55,010	0,0163	0,0167
50,864	0,0195	0,0195	56,993	0,0184	0,0193
51,967	0,0219	0,0219	58,219	0,0208	0,0215
53,006	0,0243	0,0248	59,358	0,0232	0,0239
53,769	0,0266	0,0272	60,196	0,0254	0,0260
54,505	0,0288	0,0300	61,008	0,0276	0,0283
55,219	0,0310	0,0331	61,798	0,0298	0,0308
55,717	0,0332	0,0355	62,288	0,0317	0,0326
55,708	0,0355	0,0355	62,554	0,0336	0,0336
$12\varnothing16$			$12\varnothing18$		
M, kN	χ, m^{-1}	χ, m^{-1} on the basis of equality (8) $B_0 = 5459{,}38\ kN\cdot m^2$ $\eta = 1{,}2017$; $m = 11{,}3376$	M, kN	χ, m^{-1}	χ, m^{-1} on the basis of equality (8) $B_0 = 6563{,}07\ kN\cdot m^2$ $\eta = 1{,}0662$; $m = 10{,}5669$
0	0	0	0	0	0
11,097	0,0019	0,0020	12,525	0,0018	0,0019
20,788	0,0037	0,0038	23,531	0,0035	0,0036
29,480	0,0054	0,0054	33,471	0,0051	0,0051
37,428	0,0070	0,0069	42,626	0,0067	0,0065
44,804	0,0086	0,0082	51,183	0,0083	0,0078
51,727	0,0101	0,0096	59,271	0,0098	0,0091
58,199	0,0117	0,0112	66,985	0,0112	0,0105
63,221	0,0133	0,0130	73,944	0,0127	0,0123
66,552	0,0152	0,0148	79,122	0,0144	0,0142
69,794	0,0171	0,0176	83,016	0,0163	0,0164
72,522	0,0191	0,0210	86,834	0,0181	0,0195
73,858	0,0213	0,0231	89,849	0,0199	0,0231
74,888	0,0235	0,0251	91,100	0,0220	0,0249
75,891	0,0256	0,0273	92,323	0,0241	0,0269
76,756	0,0277	0,0295	93,025	0,0260	0,0282
77,075	0,0295	0,0304	93,379	0,0278	0,0289
77,365	0,0312	0,0312	93,704	0,0295	0,0295

Also, for clarity, Graph 1 shows a comparison of graphs corresponding to each percentage of reinforcement. It is clear from these graphs that the proposed approximation makes tabular diagrams approximate with almost engineering accuracy in the entire field of variations. In these graphs, odd numbers refer to tabular diagrams, and even numbers consistently refer to the corresponding analytical approximation dependence.



Graph 1 – Comparison of tabular and analytical approximation of "moment-curvature" diagrams

And now we apply the approximation of the "moment-curvature" diagram in the form of (8) to the determination of the maximum deflection under load of a statically solvable beam sitting on two simple supports with a regular distributed load. As is known from the course of resistance of materials, the curve of a given cross-section of any bending system is determined by the Mor formula as follows [9,10]:

$$\Delta_k = \int \chi(x) \cdot \overline{M}(x) dx$$

Where $\chi(x)$ is the value of the curvature in an arbitrary x section from an external load acting on the system, and $\overline{M}(x)$ is the expression of the bending moment arising in the system from a uniform load applied to the desired section k in the direction of the desired curve. Bending moment:

$$M(x) = \frac{ql^2}{2} \cdot (\xi - \xi^2); \qquad \xi = \frac{x}{l}$$

in an arbitrary section of a simple beam sitting on articulated supports loaded with a normal load. From the unit force applied in the middle of the beam, the bending moment in the left half of the beam $\overline{M}(x) = \frac{l}{2} \cdot \xi$, then, applying the Mor formula based on approximation (8), we can write for the maximum curve of the beam that:

$$f_{\max} = 2 \cdot \int_0^{l/2} \chi(x) \cdot \overline{M}(x) dx = \frac{ql^4}{2 \cdot B_0} \cdot \left[\int_0^{1/2} (\xi - \xi^2) \cdot \xi d\xi \right] + \frac{ql^4}{2 \cdot B_0} \cdot \eta \cdot \left(\frac{ql^2}{2 \cdot M_u} \right)^m \cdot \int_0^{1/2} (\xi - \xi^2)^{m+1} \cdot \xi d\xi$$

After calculating the integral from the first table for the maximum deflection, the following expression is obtained:

$$f_{\max}=\frac{ql^4}{B_0}\cdot\left[\frac{5}{384}+\frac{\eta}{2}\cdot\left(\frac{ql^2}{2\cdot M_u}\right)^m\cdot k_m\right] \quad (13)$$

Where

$$k_m=\int_0^{1/2}\left(\xi-\xi^2\right)^{m+1}\cdot\xi d\xi \quad (14)$$

If we take into account that the expression obtained above for the maximum deflection $M_u=\frac{q_u l^2}{8}$ for a simple beam can be written in a simpler figure below:

$$f_{\max}=\frac{5\cdot ql^4}{384\cdot B_0}\cdot\left(1+\frac{192}{5}\cdot\eta\cdot\left(\frac{q}{q_u}\right)^m\cdot k_m\cdot 4^m\right) \quad (15)$$

In the limit state is $q=q_u$

$$f_{\max}=\frac{5\cdot ql^4}{384\cdot B_0}\cdot\left(1+\frac{192}{5}\cdot\eta\cdot k_m\cdot 4^m\right) \quad (16)$$

Here, the second sum takes into account the influence of non-linearity on the deflection value. For the four reinforcement options discussed above, the following values were obtained, corresponding to the amount taking into account the nonlinearity

$$\delta=\frac{192}{5}\cdot\eta\cdot k_m\cdot 4^m=0{,}7863;$$

0,7228; 0,5983; 0,5443

From this it can be seen that the maximum deflection of a simple beam, considered due to physical nonlinearity, can increase from 1.5443 to 1.7863 times.

Based on a similar technique, the following equality of the maximum deflection of the console rod was obtained

$$y_{\max}=\frac{ql^4}{8B_0}\cdot\left[1+\frac{\eta}{(2m+4)\cdot 2^{m-2}}\cdot\left(\frac{ql^2}{M_u}\right)^m\right] \quad (17)$$

Here, the second sum takes into account the increase in the maximum bending of the beam as a result of the development of plastic deformations. As you know, the limit for the console rod is the value of bending moment $M_u=\frac{q_u l^2}{2}$, so the limit in the case of the console is the maximum value of the curve of the free end of the rod

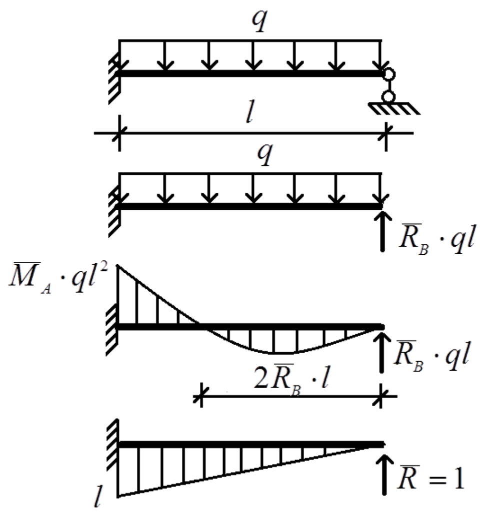
$$y_{\max,u}=y(0)=\frac{q_u l^4}{8B_0}\cdot\left(1+\frac{2\eta}{m+2}\cdot\left(\frac{q}{q_u}\right)^m\right) \quad (18)$$

In the limit state is $q=q_u$

$$y_{\max,u}=y(0)=\frac{q_u l^4}{8B_0}\cdot\left(1+\frac{2\eta}{m+2}\right) \quad (19)$$

In a static unsolvable beam one end of which is riveted, and the other end with a hinge once, when choosing the main system, as in scheme 2, $M(x)=ql^2\cdot\left(\overline{R}_B\cdot\xi-\xi^2/2\right)$ and $\overline{M}(x)=1\cdot x=l\cdot\xi$ expressions of bending moments arise from both external load and uniform force. Considering them, the expression of the Mor formula for determining the displacement of a B point will look like this:

$$\Delta_B=\int_0^l\chi(M(x))\cdot\overline{M}(x)dx=\frac{ql^4}{B_0}\cdot\int_0^1\left(\overline{R}_B\cdot\xi-\xi^2/2\right)\cdot\left[1+\eta\cdot\left(\frac{ql^2}{M_u}\right)^m\left|\overline{R}_B\cdot\xi-\xi^2/2\right|^m\right]\cdot\xi d\xi$$



Scheme 2 – Beam design diagram

As can be seen from Scheme 2, when $0 \le \xi \le 2\overline{R}_B$ then $\overline{R}_b \cdot \xi - \xi^2/2 \ge 0$ and when $2\overline{R}_B \le \xi \le 1$as $\overline{R}_b \cdot \xi - \xi^2/2 \le 0$ the above integral can be written as follows

$$\Delta_B = \frac{ql^4}{B_0} \cdot \int_0^1 \left(\overline{R}_B \cdot \xi - \xi^2/2\right) \cdot \xi d\xi + \frac{ql^4}{B_0} \cdot \eta \cdot \left(\frac{ql^2}{M_u}\right)^m \cdot \int_0^{2\overline{R}_B} \left(\overline{R}_B \cdot \xi - \xi^2/2\right)^{m+1} \cdot \xi d\xi -$$

$$- \frac{ql^4}{B_0} \cdot \eta \cdot \left(\frac{ql^2}{M_u}\right)^m \cdot \int_{2\overline{R}_B}^1 \left(\xi^2/2 - \overline{R}_B \cdot \xi\right)^{m+1} \cdot \xi d\xi$$

Here, the first sum represents the linear system and can be easily calculated as a table integral, the following non-linear equation for the determination of the unknown $\overline{R}_B$ support reaction was obtained by calculating the remaining two integrals by applying the rule of multiplication of epurs [10] of the construction mechanics course

$$\frac{\overline{R}_B}{3} - \frac{1}{8} - \eta \cdot \left(\frac{ql^2}{M_u}\right)^m \cdot \left[\frac{\left(1 - 4\overline{R}_B^2\right)^{m+2}}{3 \cdot 8^{m+1}} + \frac{\left(1 - 2\overline{R}_B\right)^{m+2}}{6 \cdot 2^{m+1}} - \frac{4}{3} \cdot \frac{\overline{R}_B^{2m+4}}{2^{m+1}}\right] = 0 \qquad (20)$$

From the obtained equation, it can be seen that if nonlinearity is not taken into account, that is when $\eta = 0$, if the equation (20) is obtained from the material resistance course, the value $\overline{R}_B = 3/8$ can be obtained. The other two limits take into account the influence of physical nonlinearity on the value of the solution. To solve this equation numerically with the desired accuracy, an appropriate programm module is compiled. The change in the reaction of the support for the examples discussed above, depending on the load level as a result of calculations performed using this programm module, is shown in table 2 below. As is known from the materials resistance course, with the inclusion of plastic joints loaded with a regular distributed load, one end is rigid and the other end is attached to the hinge, the maximum bending of the beam $M_u = \dfrac{q_u l^2}{6 + 4\sqrt{2}} = \dfrac{q_u l^2}{11{,}657}$ can vary to the value:

$$q_u l^2 = M_u \cdot \left(6 + 4\sqrt{2}\right) = M_u \cdot 11{,}657 .$$

This means that the value of $\delta = \dfrac{ql^2}{M_u}$ parameter vary to 11.657. The value of the reaction of the support or the moment of support, even in extreme cases due to physical nonlinearity in each of the four options considered, differs from the value obtained during deformations within the elastic limits by only 8%. This once again shows that when calculating static unsolvable physical nonlinear systems, the main solutions of the force method can be assumed to be equal to elastic values. Nevertheless, the physical non-linearity causes a sharp increase in the value of beam deflection.

Table 2 – Dependence of the support reactions on the load level for various reinforcement schemes

$12\varnothing12$ $B_0 = 4060{,}39\ kN\cdot m^2$ $\eta = 1{,}5875$ $m = 11{,}5061$		$12\varnothing14$ $B_0 = 4501{,}20\ kN\cdot m^2$ $\eta = 1{,}4178$ $m = 10{,}6097$		$12\varnothing16$ $B_0 = 5459{,}38\ kN\cdot m^2$ $\eta = 1{,}2017$ $m = 11{,}3376$		$12\varnothing18$ $B_0 = 6563{,}07\ kN\cdot m^2$ $\eta = 1{,}0662$ $m = 10{,}5669$	
$\delta = \dfrac{ql^2}{M_u}$	$\overline{R}_B$	$\delta = \dfrac{ql^2}{M_u}$	$\overline{R}_B$	$\delta = \dfrac{ql^2}{M_u}$	$\overline{R}_B$	$\delta = \dfrac{ql^2}{M_u}$	$\overline{R}_B$
0	0,375	0	0,375	0	0,375	0	0,375
1,1657	0,3750	1,1657	0,3750	1,1657	0,3750	1,1657	0,3750
2,3314	0,3750	2,3314	0,3750	2,3314	0,3750	2,3314	0,3750
3,4971	0,3750	3,4971	0,3750	3,4971	0,3750	3,4971	0,3750
4,6628	0,3750	4,6628	0,3751	4,6628	0,3750	4,6628	0,3751
5,8285	0,3756	5,8285	0,3757	5,8285	0,3755	5,8285	0,3756
6,9942	0,3786	6,9942	0,3786	6,9942	0,3780	6,9942	0,3780
8,1599	0,3849	8,1599	0,3846	8,1599	0,3838	8,1599	0,3834
9,3256	0,3924	9,3256	0,3917	9,3256	0,3910	9,3256	0,3902
10,4913	0,3986	10,4913	0,3975	10,4913	0,3973	10,4913	0,3963
11,6570	0,3986	11,6570	0,3975	11,6570	0,3973	11,6570	0,3963

Now let's consider the determination of the function of deflection of the beam under consideration. We approximate the deflections function as follows, taking into account the kinematic boundary conditions and the condition that the bending moment at the joint is zero if the origin is taken at the hinge end:

$$y(\xi) = w_* \cdot f \cdot \varphi(\xi),$$
$$\varphi(\xi) = 2\xi^4 - 3\xi^3 + \xi\ , \qquad (21)$$
$$w_* = 3{,}8465409103\,.$$

The parameter entered with this acceptance of the deflections function expresses the maximum deflections of the beam. In the case under consideration, the differential equation of the beam in bending is written as:

$$y''(\xi) + w_0 \cdot \psi(\xi) + \eta \cdot \delta^m \cdot w_0 \cdot \psi(\xi) \cdot |\,\psi(\xi)|^m = 0 \quad (22)$$

Here, $\psi(\xi) = \overline{R}_B \cdot \xi - \xi^2 / 2$ is the function of changing the curve of the bending along the beam and $w_0 = ql^4 / B_0$ marking are included. Since it is impossible to construct an analytical solution to this differential equation, let's solve it approximately by the Bubnov-Galerkin method. Using this method, we can write the following equation to determine the unknown maximum deflection

$$w_* \cdot f \cdot \int_0^1 \varphi''(\xi)\cdot\varphi(\xi)d\xi + w_0 \cdot \int_0^1 \psi(\xi)\cdot\varphi(\xi)d\xi + \eta\cdot\delta^m\cdot w_0\cdot\int_0^1 \psi(\xi)\cdot|\,\psi(\xi)\,|^m\cdot\varphi(\xi)d\xi = 0 \qquad (23)$$

Here the first and second integrals, as well as the tabular integral can be calculated very easily. When calculating the third integral, consider that,

$$\psi(\xi)=\begin{cases}\bar{R}_B\cdot\xi - \xi^2/2; & 0 \le \xi \le 2\bar{R}_B \\ \xi^2/2 - \bar{R}_B\cdot\xi; & 2\bar{R}_B \le \xi \le 1\end{cases}$$

With this in mind, we show the third integral as the sum of two integrals

$$A_3 = \int_0^{2\bar{R}_B}\left(2\xi^4 - 3\xi^3 + \xi\right)\cdot\left(\bar{R}_B\cdot\xi - \xi^2/2\right)^{m+1}d\xi\ -$$

$$-\int_{2\bar{R}_B}^1\left(2\xi^4 - 3\xi^3 + \xi\right)\cdot\left(\xi^2/2 - \bar{R}_B\cdot\xi\right)^{m+1}d\xi$$

Thus, for the maximum curve of the beam in question, we obtain the following equality

$$f = w_0\cdot\left(-\frac{A_2}{w_*\cdot A_1} - \frac{\eta\delta^m}{w_*}\cdot\frac{A_3}{A_1}\right) = w_0\cdot\left(\frac{\frac{\bar{R}_B}{15} - \frac{1}{56}}{w_*\cdot\frac{12}{35}} + \frac{\eta\delta^m}{w_*}\cdot\frac{A_3}{\frac{12}{35}}\right) =$$

$$= w_0\cdot\left(\frac{56\bar{R}_B - 15}{288\cdot w_*} + \frac{\eta\delta^m}{w_*}\cdot\frac{35A_3}{12}\right)$$

If we now took the value of the support reaction equal to the value of the elastic limit, taking into account that the dependence of the support reaction on physical nonlinearity is weak, as we showed above, we would get the following expression for determining the maximum deflection with engineering accuracy

$$f = \frac{w_0}{48\cdot w_*}\cdot\left(1 + \eta\delta^m\cdot k_m\right) \qquad (24)$$

Here, the coefficient that takes into account the increase in deflection due to elastic-plastic deformations is calculated as follows

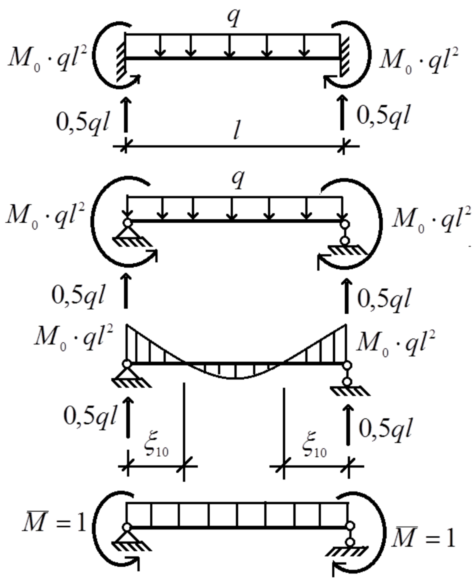
$$k_m = 140\cdot\int_0^1\left(2\xi^4 - 3\xi^3 + \xi\right)\cdot\left|\,3\xi/8 - \xi^2/2\,\right|^{m+1}d\xi \qquad (25)$$

Note that for each of the four cases considered, the following values are taken for the limit case parameter $\eta\delta^m\cdot k_m$:

$\left(\eta\delta^m\cdot k_m\right)_1 = 0{,}4459$; $\left(\eta\delta^m\cdot k_m\right)_2 = 0{,}4066$;

$\left(\eta\delta^m\cdot k_m\right)_3 = 0{,}3265$; $\left(\eta\delta^m\cdot k_m\right)_4 = 0{,}2935$.

By analogy with a riveted beam at both ends, scheme 3. Under the action of a uniform distributed load, this beam from the symmetry condition once becomes a statically insoluble system, scheme 3.



Scheme 3 – Beam design diagram

From the condition that the angle of rotation of the support is zero taking into account symmetry, the following equality was obtained to determine the reference moment:

$$\int_0^{1/2}\left(\frac{\xi}{2}-\frac{\xi^2}{2}-\overline{M}_0\right)\cdot d\xi+\eta\cdot\left(\frac{ql^2}{M_u}\right)^m\cdot$$

$$\cdot\int_0^{1/2}\left(\frac{\xi}{2}-\frac{\xi^2}{2}-\overline{M}_0\right)\cdot\left|\frac{\xi}{2}-\frac{\xi^2}{2}-\overline{M}_0\right|^m\cdot d\xi=0$$

By entering the designation of the limit of the integral, taking into account the physical nonlinearity, we obtain the following nonlinear equation for determining the moment of support:

$$k_m(\overline{M}_0)=\int_0^{1/2}\left(\frac{\xi}{2}-\frac{\xi^2}{2}-\overline{M}_0\right)\cdot\left|\frac{\xi}{2}-\frac{\xi^2}{2}-\overline{M}_0\right|^m\cdot d\xi \qquad (26)$$

$$\frac{1}{24}-\frac{\overline{M}_0}{2}+\eta\cdot\left(\frac{ql^2}{M_u}\right)^m\cdot k_m(\overline{M}_0)=0 \qquad (27)$$

As can be seen when $\eta=0$ then from equation (27), we obtain the value $\overline{M}_0=1/12$ known from the resistance rate of materials. Based on the equation of the general case (27), a programm module is compiled that implements the definition of the reference moment parameter. To determine the maximum deflection in the middle of the beam in question, the following formula was obtained, acting as indicated above:

$$f_{\max}=\frac{ql^4}{B_0}\cdot\left\{\frac{48\overline{M}_0-3}{384}+\frac{1}{12}\cdot\eta\cdot\left(\frac{ql^2}{M_u}\right)^m\cdot\left[\frac{\overline{M}_0}{2}\cdot\left|\overline{M}_0\right|^m-\left(\frac{3}{32}-\overline{M}_0\right)\cdot\left|\frac{3}{32}-\overline{M}_0\right|^m\right]\right\} \qquad (28)$$

The maximum deflection of the beam in the middle of the span, considered within the elastic limit, $(f_{\max})_{el}=\dfrac{ql^4}{384B_0}$ is known from the course of resistance of materials. Add this value to the right side of the above equality and subtract, then we get that,

$$f_{\max}=\frac{ql^4}{B_0}\cdot\left\{\frac{1}{384}+\frac{12\overline{M}_0-1}{96}+\frac{1}{12}\cdot\eta\cdot\left(\frac{ql^2}{M_u}\right)^m\cdot\left[\frac{\overline{M}_0}{2}\cdot\left|\overline{M}_0\right|^m-\left(\frac{3}{32}-\overline{M}_0\right)\cdot\left|\frac{3}{32}-\overline{M}_0\right|^m\right]\right\}$$

or in a simpler way:

$$f_{\max}=\frac{ql^4}{384B_0}\cdot(1+f_m) \qquad (29)$$

Here is a coefficient that takes into account the influence of nonlinearity on the maximum deflection of the beam

$$\lambda_M=32\cdot\left[\frac{\overline{M}_0}{2}\cdot\left|\overline{M}_0\right|^m-\left(\frac{3}{32}-\overline{M}_0\right)\cdot\left|\frac{3}{32}-\overline{M}_0\right|^m\right];\qquad f_m=48\overline{M}_0-4+\eta\cdot\left(\frac{ql^2}{M_u}\right)^m\cdot\lambda_M \qquad (30)$$

When using the above programm module, the value of the maximum deflection resulting from non-linearity in the beam under consideration was investigated, and its results are shown in the table below.

Table 3 – Influence of non-linearity on the value of maximum deflections of beams

$12\varnothing 12$; $B_0 = 4060{,}39\ kN\cdot m^2$; $\eta = 1{,}5875$; $m = 11{,}5061$			$12\varnothing 14$; $B_0 = 4501{,}20\ kN\cdot m^2$; $\eta = 1{,}4178$; $m = 10{,}6097$		
$\delta = \frac{ql^2}{M_u}$	$\overline{M}_0$	f_m	$\delta = \frac{ql^2}{M_u}$	$\overline{M}_0$	f_m
0	0,0833	0	0	0,0833	0
1,6	0,0833	0	1,6	0,0833	0
3,2	0,0833	0	3,2	0,0833	0
4,8	0,0833	0	4,8	0,0833	0,0001
6,4	0,0833	0,0014	6,4	0,0833	0,0022
8,0	0,0833	0,0183	8,0	0,0833	0,0234
9,6	0,0831	0,1444	9,6	0,0830	0,1562
11,2	0,0821	0,7327	11,2	0,0821	0,6985
12,8	0,0796	2,3074	12,8	0,0797	2,0603
14,4	0,0758	4,9434	14,4	0,0763	4,3327
16,0	0,0721	8,9630	16,0	0,0728	7,7673
$12\varnothing 16$; $B_0 = 5459{,}38\ kN\cdot m^2$; $\eta = 1{,}2017$; $m = 11{,}3376$			$12\varnothing 18$; $B_0 = 6563{,}07\ kN\cdot m^2$; $\eta = 1{,}0662$; $m = 10{,}5669$		
$\delta = \frac{ql^2}{M_u}$	$\overline{M}_0$	f_m	$\delta = \frac{ql^2}{M_u}$	$\overline{M}_0$	f_m
0	0,0833	0	0	0,0833	0
1,6	0,0833	0	1,6	0,0833	0
3,2	0,0833	0	3,2	0,0833	0
4,8	0,0833	0	4,8	0,0833	0,0001
6,4	0,0833	0,0012	6,4	0,0833	0,0017
8,0	0,0833	0,0148	8,0	0,0833	0,0179
9,6	0,0831	0,1143	9,6	0,0831	0,1197
11,2	0,0823	0,5823	11,2	0,0823	0,5472
12,8	0,0801	1,9043	12,8	0,0803	1,6896
14,4	0,0767	4,2289	14,4	0,0771	3,6952
16,0	0,0730	7,6544	16,0	0,0736	6,6462

It can be seen from the table that the value of the maximum deflection differs by less than 0.155% from the value set within the elastic limit, since the development of plastic deformations is weak when the limit in the example under consideration does not exceed half the value of the load on a statically insoluble rigid beam twice, but with a further increase in the impact load, plastic deformations begin to develop intensively, as a result, the value of the maximum deflection of the beam begins. The deflection value resulting from the ultimate plasticity, within the elastic limit, can be several times higher than the value determined by the formula of the resistance of materials. This table also shows that an increase in the percentage of reinforcement leads to a decrease in the value of maximum deflection. Please note that the influence of the percentage of reinforcement on the value of the maximum deflection is not so great, and, for example, less than 20%.

Conclusion

An effective numerical technique for constructing a “moment-curvature” diagram for reinforced concrete elements of annular cross-section has been developed.

For reinforced concrete elements of annular cross-section, an analytical approximation of the “moment-curvature” diagram with high accuracy is proposed.

For reinforced concrete elements of annular cross-section, the corresponding analytical formulas for determining the maximum deflections are proposed.

Conflict of Interests

The authors declare there is no conflict of interest related to the publication of this article.

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